



Norwegian University of Science
and Technology
Department of Mathematical
Sciences

TMA4145 Linear Methods
Fall 2022

Exercise set 12

1 Let $\{e_n\}_{n \in \mathbb{N}}$ be the standard basis in the sequence space l^∞ . Show that the series $\sum_{n=1}^{\infty} a_n e_n$ converges in l^∞ if and only if $\{a_n\}_{n \in \mathbb{N}}$ converges to zero.

2 Show that if a normed space $(X, \|\cdot\|)$ has a Schauder basis, then it is separable.

3 Let $L^2[-1, 1]$ be equipped with the inner product

$$\langle f, g \rangle = \int_{-1}^1 f(t) \overline{g(t)} dt.$$

Apply Gram-Schmidt's orthogonalization algorithm to the monomial basis $\{1, x, x^2, \dots\}$ up to degree 2.

4 Let $\|\cdot\|_a$ and $\|\cdot\|_b$ be equivalent norms on a vector space X . Show that a sequence $\{x_n\}$ in X is Cauchy with respect to the norm $\|\cdot\|_a$ if and only if it is Cauchy with respect to the norm $\|\cdot\|_b$.