Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4145 Linear Methods
Fall 2022

**Exercise set 11**

1 Let $M$ be a subspace of an inner product space $X$. Show that the orthogonal complement $M^{\perp}$ is closed.

2 Let M be the plane of $\mathbb{R}^3$ given by $x_1 + x_2 + x_3 = 0$. Find the linear mapping that is the orthogonal projection of $\mathbb{R}^3$ onto this plane.

3 Let $T : X \to X$ be a bounded linear operator on a Hilbert space $X$. Show that

$$||TT^*|| = ||T^*T|| = ||T||^2.$$

4 Let $M$ be a closed subspace of a Hilbert space $X$, which by the projection theorem is given by the direct sum $X = M \oplus M^{\perp}$. Show that the projection onto $M$ is self-adjoint.

5 Show that $\{e^{2\pi int}\}_{n\in\mathbb{Z}}$ is orthonormal with respect to the inner product

$$\langle f, g\rangle = \int_0^1 f(t)\overline{g(t)}\, dt.$$