Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4145 Linear Methods

Fall 2020

Exercise set 12

Please justify your answers! Note that *how* you arrive at an answer is more important than the answer itself.

1 Suppose that $v_1, \dots, v_k$ are non-zero eigenvectors of an operator T corresponding to distinct eigenvalues $\lambda_1, \dots, \lambda_k$. Show that $\{v_1, \dots v_k\}$ is a linearly independent set.

2 Let T be the shift operator on ℓ^2 defined by $T(x_1, x_2, ...) = (0, x_1, x_2, ...)$.

1. Show that T has no eigenvalues.
2. Does T^* have any eigenvalues?

3 Let U be a $n \times n$ matrix with columns $u_1, ..., u_n$. Show that the following statements are equivalent:

1. U is unitary.
2. $\{u_1, ..., u_n\}$ is an orthonormal basis of $\mathbb{C}^n$.

4 Suppose that A and B are *unitarily equivalent*, meaning that there exists a unitary matrix U such that

$$B = U^* A U.$$

Prove that A is positive definite (semi-definite) if and only if B is positive definite (semi-definite).

5 Given the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \\ 2 & 1 \end{pmatrix}.$$

a) Compute the singular value decomposition of A.

b) Use the result of a) to find:

1. The pseudo-inverse of A.
2. The minimal norm solution of $Ax = b$ for $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.