Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4145 Linear Methods
Fall 2019

Exercise set 11

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

1 Consider the exponential basis $\{e^{2\pi int} : n \in \mathbb{Z}\}$ in $(L^2[0,1], \langle ., . \rangle)$. Show that these are orthonormal vectors.

2 Let $\{e_n : n \in \mathbb{N}\}$ be the standard basis in the sequence space ℓ^p.

a) Show that the series $\sum_{n=1}^{\infty} \alpha_n e_n$ converges in ℓ^p for $1 \leq p < \infty$ if and only if $(\alpha_n)_{n\in\mathbb{N}} \in \ell^p$.

b) Show that the series $\sum_{n=1}^{\infty} \alpha_n e_n$ converges in ℓ^∞ if and only if $(\alpha_n)_{n\in\mathbb{N}}$ converges to zero.

3 Show that if a normed space $(X, \|\cdot\|)$ has a Schauder basis, then it is separable.

4 Let $L^2[-1,1]$ be equipped with the inner product

$$\langle f, g \rangle = \int_{-1}^{1} f(t)\overline{g(t)}dt.$$

Apply Gram-Schmidt's orthogonalization algorithm to the monomial basis $\{1, x, x^2, x^3, ...\}$ up to degree 2.

5 *(Exam 2014, Problem 3)*

a) Let $C([0,2] \times [0,2], \mathbb{R})$ be an inner-product space with

$$\langle f, g \rangle = \int_0^2 \int_0^2 f(x,y)g(x,y)\,dx\,dy.$$

Find an orthogonal basis for $\text{span}\{1, x, y\}$ in this space.

b) Find $a, b, c \in \mathbb{R}$ such that

$$\int_0^2 \int_0^2 |xy - a - bx - cy|^2 \, dx \, dy$$

is minimal.

6 Let $\|\cdot\|_a$ and $\|\cdot\|_b$ be equivalent norms on a vector space X. Show that a sequence (x_n) in X is Cauchy with respect to the norm $\|\cdot\|_a$ if and only if it is Cauchy with respect to the norm $\|\cdot\|_b$.