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Exercise set 1

Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

1 Let X, Y and Z be sets.

a) Show that $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$.

b) Show that $X \backslash (Y \cup Z) = (X \backslash Y) \cap (X \backslash Z)$.

2 Define functions on $\mathbb{R}$ with values in $\mathbb{R}$:

i) A function that is not left invertible;

ii) A function that is not right invertible.

Show that the given functions have their respective properties.

3 Given the linear mapping $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T = Ax$ with

$$A = \begin{pmatrix} -3 & -4 \\ 4 & 6 \\ 1 & 1 \end{pmatrix}.$$

a) Show that the matrix

$$A_l^{-1} = \frac{1}{9} \begin{pmatrix} -11 & -10 & 16 \\ 7 & 8 & -11 \end{pmatrix}$$

induces a left inverse T_l^{-1} of T.
This left inverse is not unique. Show that

$$\frac{1}{2} \begin{pmatrix} 0 & -1 & 6 \\ 0 & 1 & -4 \end{pmatrix}$$

gives another left inverse.

b) Turn this example into one for right inverses. Concretely, find a mapping $S : \mathbb{R}^3 \to \mathbb{R}^2$ that is based on the mapping T and give a right inverse for this mapping.

4 Show that the Cartesian product of two (infinite) countable sets is countable.

5 Show that the sets $\mathbb{Z}$ of integers and $\mathbb{Q}$ of rational numbers are countable.