



Please justify your answers! The most important part is *how* you arrive at an answer, not the answer itself.

- 1 Let T be the cyclic (left) shift on \mathbb{R}^3 . That is, the linear operator $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$$T(x_1, x_2, x_3) = (x_2, x_3, x_1).$$

If $p \in \mathcal{P}_2$, say $p(t) = a_0 + a_1t + a_2t^2$, we can define a new operator

$$p(T) = a_0I + a_1T + a_2T^2. \quad (1)$$

Let

$$\mathcal{C} = \{p(T) : p \in \mathcal{P}_2\}$$

be the collection of all such operators.

- Describe the matrices representing operators in \mathcal{C} .
 - Show that the sum and product of two operators in \mathcal{C} is again in \mathcal{C} , and that they commute.
(Two operators A, B commute if $AB = BA$; that is, their product does not depend on the order.)
 - In problem set 5, you determined the eigenvalues and eigenvectors of T (on \mathbb{C}^3). Use this to diagonalize all members of \mathcal{C} .
- 2 Let T be a nilpotent operator on a finite-dimensional vector space V . Show that $I - T$ is invertible, and give a formula for its inverse. Apply this result to the differentiation operator D on \mathcal{P}_3 (that is, $(Dp)(x) = p'(x)$).

- 3 Let A be the matrix

$$A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ 1 & -1 & 4 \end{bmatrix}.$$

- Compute the eigenvalues of A . What are their algebraic and geometric multiplicities?
- Find a Jordan normal form PJP^{-1} of A .
(You do not have to explicitly invert the P you find.)