TMA4140
DISKRET MATEMATIKK – DISCRETE MATHEMATICS
NTNU, HØST/FALL2020

SOLUTIONS – EXERCISE SET 1

Exercise/Oppgave

1. Section/Sektion 1.1: *14b,c,e, 16a,b,f, 34d,f*

Solution.

- 14 b) You do not miss the final examination if and only if you pass the course.
  14 c) If you miss the final examination, then you do not pass the course.
  14 e) If you have the flu then, you do not pass the course, or, if you miss the final examination, then you do not pass the course.
- 16 a) $r \wedge \neg q$.
  16 b) $p \wedge q \wedge r$.
  16 f) $r \Leftrightarrow (q \vee p)$
- 34 d)

p	q	$p \wedge q$	$p \vee q$	$p \wedge q \Rightarrow p \vee q$
1	1	1	1	1
1	0	0	1	1
0	1	0	1	1
0	0	0	0	1

  34 f)

p	q	$\neg q$	$p \Leftrightarrow q$	$p \Leftrightarrow \neg q$	$(p \Leftrightarrow q) \oplus (p \Leftrightarrow \neg q)$
1	1	0	1	0	1
1	0	1	0	1	1
0	1	0	0	1	1
0	0	1	1	0	1

□

Exercise/Oppgave

2. *Negate/Bestem negasjonen av:* $p \to (\neg q \leftrightarrow r)$

Solution. Using Laws of Logic, we have that

$$\begin{aligned}
\neg(p \to (\neg q \leftrightarrow r)) &\equiv p \wedge \neg(\neg q \leftrightarrow r) \\
&\equiv p \wedge \neg((\neg q \to r) \wedge (r \to \neg q)) && \text{by definition of } \leftrightarrow \\
&\equiv p \wedge (\neg(\neg q \to r) \vee \neg(r \to \neg q)) && \text{by DeMorgan's Law} \\
&\equiv p \wedge ((\neg q \wedge \neg r) \vee (r \wedge \neg(\neg q))) \\
&\equiv p \wedge ((\neg q \wedge \neg r) \vee (r \wedge q)) && \text{by Law of Double Negation}
\end{aligned}$$

□

Date: August 23, 2020.

Exercise/Oppgave

3. Section/Sektion 1.3: *12, 38c*

Solution.

- 12 a)

p	q	$p \vee q$	$\neg p \wedge (p \vee q)$	$(\neg p \wedge (p \vee q)) \to q$
1	1	1	0	1
1	0	1	0	1
0	1	1	1	1
0	0	0	0	1

12 b)

p	q	r	$q \to r$	$p \to q$	$p \to r$	$(q \to r) \wedge (p \to q)] \to (p \to r)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	0	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	0	1
1	0	1	1	0	1	1
1	1	0	0	1	0	1
1	1	1	1	1	1	1

12 c)

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
1	1	1	1	1
1	0	0	0	1
0	1	1	0	1
0	0	1	0	1

12 d)

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
0	1	0	1	1	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Since the last columns of the tables are only 1's, we conclude that the four of the propositions are tautologies.

- 38 c) Recall that the dual is constructed by interchanging $\vee$ by $\wedge$, $\wedge$ by $\vee$, F by T, and T by F. We do not change $\neg$. Hence, if $s = (p \wedge \neg q) \vee (q \wedge \mathbf{F})$, then $s^* = (p \vee \neg q) \wedge (q \vee \mathbf{T})$.

□

Exercise/Oppgave

4. *Use the laws of logic to simplify/Bruk logikkens lover til å forenkle:*

$$\big(p \lor (p \land q) \lor (p \land q \land \neg r)\big) \land \big((p \land r \land t) \lor t\big)$$

Solution. Notice that, by Absorption Law, $(p \land r \land t) \lor t \equiv t$. Also by Associative and Absorption Law,

$$\big(p \lor (p \land q)\big) \lor (p \land q \land \neg r) \equiv p \lor (p \land q \land \neg r) \equiv p.$$

Hence

$$\big(p \lor (p \land q) \lor (p \land q \land \neg r)\big) \land \big((p \land r \land t) \lor t\big) \equiv p \land t.$$

□

Exercise/Oppgave

5. Section/Sektion 1.4: *24d,e.*

Solution.

- 24 d) Let $P(x) = x$ can solve quadratic equations. If the domain consists of the students of the class, then the statement can be translated as

$$\forall x\, P(x).$$

  On the other hand, if the domain consists in all people, if we define $Q(x) = x$ is a student of the class, then the statement can be translated as

$$\forall x\, (Q(x) \to P(x)).$$

- 24 e) Let $P(x) = x$ wants to be rich, and $Q(x) = x$ is a student of the class. In the first case that the domain consists of the students of the class, since there exists one student in the class that does not want to be rich, this can be translated as

$$\exists x\, \neg P(x).$$

  On the other hand, if the domain consists in all people, we also have the additional condition that x is a student of the class. Then, the statement can be translated as

$$\exists x\, (Q(x) \land \neg P(x)).$$

□

Exercise/Oppgave

6. Section/Sektion 1.5: *12b,e,j, 28d,h, 30b,c,e*

Solution.

- 12 b) Define $C(x, y)$ as the statement "x and y have chatted over the Internet". Then the statement can be expressed as $\neg C(\text{Rachel}, \text{Chelsea})$.
- 12 e)

$$(\neg C(\text{Sanjay}, \text{Joseph})) \land (\forall\, x \neq \text{Joseph}\ C(\text{Sanjay}, x)).$$

- 12 j) Let $I(x)$ be the statement "x has an Internet connection". The desired statement can be expressed as

$$\forall x\, (I(x) \to \exists y((y \neq x) \land (C(x, y)))).$$

- 28 d,h) Let $\mathbb{R}$ be the set of real numbers. We know that the addition of real numbers satisfies the commutative property, which states that for any two real numbers x,y, we have that $x + y = y + x$. Hence, the truth value of the expression $\exists x \exists y(x + y \neq y + x)$ is false. Now consider the proposition $\exists x \exists y(x + 2y = 2 \wedge 2x + 4y = 5)$. This proposition is equivalent to say that the the system of equations $x + 2y = 2, 2x + 4y = 5$ has a solution. However, we can easily see that the system has no real solutions. Hence, the truth value of the proposition is false.
- 30) Recall that $\neg \exists x P(x) \equiv \forall x \neg P(x)$ and $\neg \forall x P(x) \equiv \exists x \neg P(x)$. For the item b), we have

$$\begin{aligned} \neg \forall x \exists y P(x, y) &\equiv \exists x \neg(\exists y P(x, y)) \\ &\equiv \exists x \forall y \, \neg P(x, y). \end{aligned}$$

For c), by using DeMorgan's law we have

$$\begin{aligned} \neg \exists y(Q(y) \wedge \forall x \neg R(x, y)) &\equiv \forall y \neg(Q(y) \wedge \forall x \neg R(x, y)) \\ &\equiv \forall y(\neg Q(y) \vee \neg \forall x \neg R(x, y)) \\ &\equiv \forall y(\neg Q(y) \vee \exists x \neg\neg R(x, y)) \\ &\equiv \forall y(\neg Q(y) \vee \exists x R(x, y)), \end{aligned}$$

by double negation. Finally, for e) we have

$$\begin{aligned} \neg \exists y(\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z)) &\equiv \forall y \neg(\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z)) \\ &\equiv \forall y(\neg \forall x \exists z T(x, y, z) \wedge \neg \exists x \forall z U(x, y, z)) \\ &\equiv \forall y(\exists x \neg \exists z T(x, y, z) \wedge \forall x \neg \forall z U(x, y, z)) \\ &\equiv \forall y(\exists x \forall z \neg T(x, y, z) \wedge \forall x \exists z \neg U(x, y, z)). \end{aligned}$$

□