TMA4140
DISKRET MATEMATIKK – DISCRETE MATHEMATICS
NTNU, HØST/FALL2020

EXERCISE SET 12 / ØVING 12

The solutions must be submitted via OVSYS (to the assigned group/TA).
Løsningene må sendes inn via OVSYS (til den tildelte gruppen/TA).

Deadline for submission: **Friday, 27 November, 4:30pm**
Innleveringsfrist: **Fredag, 27. november, kl. 16:30**

Textbook: K. H. Rosen, *Discrete Mathematics and Its Applications*, 8. edition

Exercise 1. *Check whether* $(p \to (q \to r)) \to ((p \wedge q) \to r)$ *is a tautology.*

Exercise 2. *Let* X, Y, Z *be sets. Use the laws of set theory to show that*

$$(X \cup Y \cup Z) \triangle (X \cap Y \cap Z) = (X \cup Y \cup Z) \setminus (X \cap Y \cap Z).$$

Exercise 3. *Let* $X = \{1, 2, 3, 4\}$. *Consider the relation*

$$R = \{(1,1), (1,2), (1,4), (2,3), (3,1), (3,4), (4,2), (4,4)\}$$

Determine the closures regarding: reflexive, symmetric, and transitive properties of R.

Exercise 4. *i) Consider the function* f *with the natural numbers as domain and codomain, which maps a number written in decimal representation to the sum of squares of its decimal digits. For example, for* $n = 321$, $f(321) = 3^2 + 2^2 + 1^2$. *Determine whether* f *is injective, surjective, or both.*

ii) Show that for positive n, *the function* $f(n) := \sum_{i=1}^{n} i^2$ *is* $\Omega(n^3)$.

Exercise 5. *i) Use induction to show for* n *a positive integer that* $(n+2)^{n+3} > (n+3)^{n+2}$.

ii) Recall the definition of the Harmonic numbers H_n. *Use induction to show that*

$$\sum_{j=1}^{n} \frac{H_{j+1}}{j(j+1)} = 2 - \frac{1}{n+2} - \frac{H_{n+2}}{n+1}.$$

Exercise 6. *Compute the coefficient of* x^3 *in the expansion* $(x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^{\frac{1}{5}})^{20}$.

Exercise 7. *Find a solution to the recurrence relation* $a_n = -a_{n-1} + 2a_{n-2} + 2a_{n-3}$, *for* $n > 3$ *and* $a_1 = a_2 = a_3 = 3$.

Exercise 8. *Consider* $\mathbb{Z}_7$ *and prove that for all* $[a] \in \mathbb{Z}_7$, $[a] \neq [0]$ *we have that* $[a]^6 = [1]$. *Now, let* n *be a positive integer and assume that* $gcd(n, 7) = 1$. *Verify that* 7 *divides* $n^6 - 1$.

Date: November 19, 2020.

Exercise 9. *We define for positive* n *the so-called prism graphs* p_n*. In Figure 1 you see* p_4*. The number* T_{n+1} *of spanning trees in* p_{n+1} *is given by* $T_{n+1} = 2T_n + T_{n-1} + \cdots + T_1 + 1$*. Verify this statement for* $n = 4$*. Show for* $n > 1$ *that* $T_{n+1} = 3T_n - T_{n-1}$*. Solve this recurrence relation for* T_n*,* $n > 0$ *and find the link to the Fibonacci numbers.*

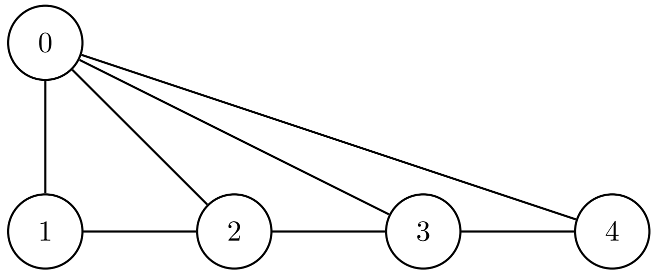

FIGURE 1. Prism graph p_4.

Exercise 10. *a) i) What is the state table* $T(A)$ *for the automaton* A *in Figure 2?*

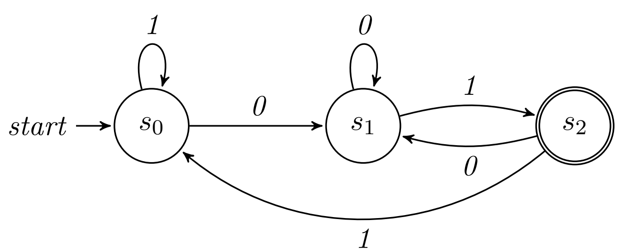

FIGURE 2. The automaton A.

ii) Determine the arrival state for each of the input sequences

$a)$ 01 $b)$ 0011 $c)$ 010101

iii) What is the language $L(A)$ *accepted by* A*?*

b) *Find the language* $L(A)$ *accepted by the automaton* A' *in Figure 3.*

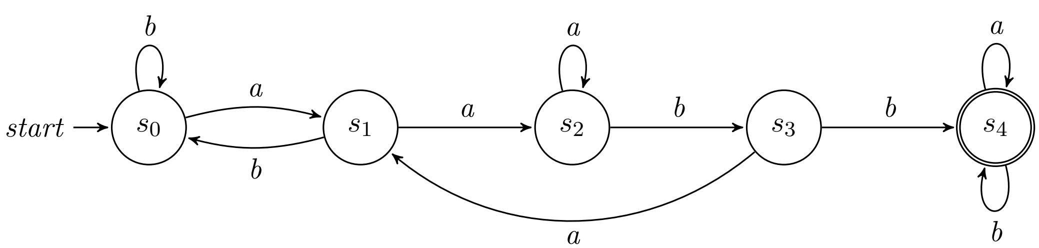

FIGURE 3. The automaton A'.

Exercise 11. Section/Sektion 13.1: *Please read this section at home.*

Exercise 12. Section/Sektion 13.2: *2 a, 4 a*

Exercise 13. Section/Sektion 13.3: *8, 10, 12, 16, 22, 24, 36*