TMA4140
DISKRET MATEMATIKK – DISCRETE MATHEMATICS
NTNU, HØST/FALL2020

EXERCISE SET 9 / ØVING 9

The solutions must be submitted via OVSYS (to the assigned group/TA).
Løsningene må sendes inn via OVSYS (til den tildelte gruppen/TA).

Deadline for submission: **Friday, 30 October, 4:00pm**
Innleveringsfrist: **Fredag, 30. Oktober, kl. 16:00**

Textbook: K. H. Rosen, *Discrete Mathematics and Its Applications*, 8. edition

Exercise/Oppgave
1. *Consider the set* $X = \{2, 16, 128, 1024, 8192, 65536\}$*. Use the pigeonhole principle to show that if four numbers are selected from* X*, then two of those four numbers must have the product* 131072*. Hint: think in terms of powers of* 2*.*

Exercise/Oppgave
2. *Use induction to show that* $\sum_{k=1}^{n}(6k-4) = n(3n-1)$*.*

Exercise/Oppgave
3. *Consider the sequence* $\{f_n\}_{n\geq 0}$ *of Fibonacci numbers determined for* $n > 1$ *by* $f_n = f_{n-1} + f_{n-2}$, $f_0 = 0, f_1 = 1$*. Show by induction that for positive integers* n*:* $\sum_{i=1}^{n}(-1)^{i+1}f_{i+1} = (-1)^{n-1}f_n$*.*

Exercise/Oppgave
4. *Let* S *be a set and let* $P = \{A_1, \ldots, A_k\}$ *be a partition of* S*. We define the map* $f : S \to P$ *by* $f(s) = A_j$ *if* $s \in A_j$*. Show that* f *is surjective.*

Exercise/Oppgave
5. *Let* X *be a non-empty set and consider functions* $f, g : X \to X$*. Assume that* $f = g \circ f \circ f$ *and* $g = f \circ g \circ f$*. Show that* $f = g$*.*

Exercise/Oppgave
6. *Let* $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$*. Consider the set partitions* P_1 *of* A *with blocks* $P_{11} = \{1, 3, 5, 7, 9\}$, $P_{12} = \{2, 4, 6, 8\}$*, and the set partition* P_2 *of* A *with blocks* $P_{21} = \{1, 2, 3, 4\}$, $P_{22} = \{5, 7\}$, $P_{23} = \{6, 8, 9\}$*. Compute the set* $P_3 := \{P_{1i} \cap P_{2j} \mid i = 1, 2,\ j = 1, 2, 3\} \backslash \emptyset$*. Show that* P_3 *is a partition of* A*.*

Date: October 21, 2020.

Exercise/Oppgave

7. *Consider a surjective function* $f : A \to B$. *Define for* $b \in B$ *the set* $f^{-1}(b) := \{a \in A \mid f(a) = b\} \subseteq A$. *Show that* $P := \{f^{-1}(b) \mid b \in B\}$ *defines a partition of* A.

Exercise/Oppgave

8. *Let* $A = \{2, 3, 4, 6, 9\}$. *Draw the directed graph of the relation defined by*

$$R = \{(2, 3), (2, 9), (3, 2), (3, 4), (4, 3), (4, 9), (9, 2), (9, 4)\}$$

Exercise/Oppgave

9. Section/Sektion 9.1: *2a, 7, 42a, 42c*

Exercise/Oppgave

10. Section/Sektion 9.3: *10, 14, 22*