NTNU
Norwegian University of
Science and Technology

Department of Mathematical Sciences

Midterm examination paper for **TMA4140 Diskret Matematikk**

Academic contact during examination: Sverre Smalø

Phone: 73591750/48293975

Examination date: 19. oktober 2019

Examination time (from–to): 18:15-19:45

Permitted examination support material: C: Specified printed and handwritten aids permitted, specifically only Discrete Mathematics and Its Applications by Kenneth H. Rosen. Specified simple calculator permitted.

Language: English

Number of pages: 7

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av midtsemesterprøve	
Originalen er:	
1-sidig ☐	2-sidig ☒
sort/hvit ☒	farger ☐
skal ha flervalgskjema ☐	

Date Signature

INSTRUCTIONS:

This test is a multiple choice test. The last page of the this problem set is a sheet with a table where you are meant to fill in a ”cross” or an ”x” to indicate your answers. The last page with the table of answers is to be marked with your candidate number and handed in. You should only hand in the page with the table of answers.

There will be at least one correct alternative for each problem, but there may be several. One receives 1 point for each correct alternative, while one is deducted 1 point for each incorrect alternative. There are between 15 and 30 correct alternatives.

Problem 1 Which of the following propositions are tautologies?

Alt 1) $\neg((p \to r) \wedge (\neg p \to q)) \vee (\neg q \to r)$ •

Alt 2) $(r \vee \neg(q \wedge p)) \leftrightarrow ((\neg p \vee r) \wedge (\neg q \vee r))$

Alt 3) $(\neg q \to \neg p) \wedge (\neg r \to \neg q)$

Alt 4) $(\neg p \vee (\neg r \to \neg q)) \leftrightarrow (\neg r \to \neg(p \wedge q))$ •

Problem 2 Given the recurrence relation $a_n = 10a_{n-1} - 25a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 10$, what is a_{10}?

Alt 1) $5^{10} + 2 \cdot 5^{11}$ •

Alt 2) 9390624

Alt 3) $(11001100111001000001011001 1)_2$ •

Alt 4) $(43645040)_8$

Problem 3 Let $f_1 \colon \mathbb{N} \to \mathbb{R}$ be given by $f_1(n) = (2n)!$, $f_2(n) = n^n$, $f_3(n) = e^n$ and $f_4(n) = n^4$. Which of the following claims are correct?

Alt 1) f_3 er $O(f_2)$ •

Alt 2) f_4 er $\Theta(f_3)$

Alt 3) f_3 er $O(f_1)$ •

Alt 4) f_1 er $O(f_2)$

Problem 4 Let the universal set be $\mathbb{Z}_{42} = \{0, 1, \ldots, 41\}$.

Which of the following claims are correct?

Alt 1) $\exists b \forall a (a \cdot b \equiv b \pmod{42})$ •

Alt 2) $\exists b (b \neq 1) \forall a (a^b \equiv a \pmod{42})$ •

Alt 3) The number of elements in $\mathbb{Z}_{42}$ that have an inverse modulo 42 in $\mathbb{Z}_{42}$ is 13.

Alt 4) The congruence $x^2 \equiv 1 \pmod{42}$ has exactly 4 solutions in $\mathbb{Z}_{42}$. •

Problem 5 For $i = 1, 2, 3, 4, 5$ let $x_i \in \mathbb{N}$.

How many $(x_1, x_2, x_3, x_4, x_5)$ are there in $\mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ such that $x_1 + x_2 + x_3 + x_4 + x_5 = 12$?

Alt 1) 1820 •

Alt 2) 4368

Alt 3) $(130130)_4$ •

Alt 4) $(10420)_8$

Problem 6 Consider $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ and let

$$\mathbb{Z}_6 \times \mathbb{Z}_6 \times \mathbb{Z}_6 \times \mathbb{Z}_6 \times \mathbb{Z}_6 = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in \mathbb{Z}_6, i = 1, 2, 3, 4, 5\}.$$

Let now $A = \{(1, 1, a_3, a_4, a_5) \mid a_i \in \mathbb{Z}_6, i = 3, 4, 5\}$, $B = \{(a_1, a_2, a_3, 5, 5) \mid a_i \in \mathbb{Z}_6, i = 1, 2, 3\}$ and $C = \{(a_1, 3, a_3, 4, a_5) \mid a_i \in \mathbb{Z}_6, i = 1, 3, 5\}$.

How many elements are there in $A \cup B \cup C$?

Alt 1) 643

Alt 2) $(1010000010)_2$ •

Alt 3) $(1203)_8$ •

Alt 4) $(456)_{12}$ •

Problem 7 What is the coefficient in front of the monomial x^6y^3 in the polynomial $2x^2y(3x - 5y)^6$?

Alt 1) 60750 •

Alt 2) 65966

Alt 3) 425

Alt 4) 30375

Problem 8 How many solutions of the congruence $x^2 \equiv 1 \pmod{56}$ are there in $\mathbb{Z}_{56} = \{0, 1, 2, \ldots, 55\}$?

Alt 1) 2

Alt 2) 4

Alt 3) 6

Alt 4) 8 •

Problem 9 Consider all the permutations of $\{a, b, c, d, e\}$ and order them in lexicographical order. Which permutation immediately precedes $bcade$?

Alt 1) $bcaed$

Alt 2) $bacde$

Alt 3) $baedc$ •

Alt 4) $aedcb$

Problem 10 Which of the following functions $f_i \colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ for $i = 1, 2, 3, 4$ are surjective/onto?

Alt 1) $f_1(m, n) = n^2 + n^3$

Alt 2) $f_2(m, n) = m - n$ •

Alt 3) $f_3(m, n) = 13m - 3n$ •

Alt 4) $f_4(m, n) = 8m - 2n$

Problem 11 Which of the following functions $f_i\colon \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$ for $i = 1, 2, 3, 4$ are injective?

Alt 1) $f_1(x, y) = x \cdot y$

Alt 2) $f_2(x, y) = 2^x 3^y$ •

Alt 3) $f_3(x, y) = f_2(x, y) + 3x$

Alt 4) $f_4(x, y) = 5^x - 3^y$

Problem 12 2020 is a leap year. The 17th of May was a Friday in 2019. Which day of the week is the 17th of May in 2021?

Alt 1) Saturday

Alt 2) Sunday

Alt 3) Monday •

Alt 4) Tuesday

TABLE OF ANSWERS

Fill in a ”cross” or ”x” to mark the alternatives you believe are correct. One receives 1 point per correct alternative, while one is deducted 1 point per incorrect alternative. Mark this page with your candidate number, and hand it in.

Candidate number:

	Alt 1	Alt 2	Alt 3	Alt 4
Oppgave 1				
Oppgave 2				
Oppgave 3				
Oppgave 4				
Oppgave 5				
Oppgave 6				
Oppgave 7				
Oppgave 8				
Oppgave 9				
Oppgave 10				
Oppgave 11				
Oppgave 12				