Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4135
Matematikk 4D
Fall 2019

Exercise set 1

Submission deadline: 10th September

The first two exercises based on the following;

https://wiki.math.ntnu.no/_media/tma4135/2019h/preliminaries.pdf

1 Use $e^{i\pi/2} = i$ to compute $\cos(n\pi/2)$ and $\sin(n\pi/2)$, $n = 0, 1, \cdots$.

Hint: consider n-th power of $e^{i\pi/2} = i$*, apply the Euler formula; then assume that* $n = 2k, 2k+1$.

2 Compute

$$\int_{-\pi}^{\pi} x e^{inx}\, dx, \quad n = 0, 1, \cdots .$$

Hint: $n = 0$ *is easy; for* $n \neq 0$ *use* $e^{inx} = (\frac{e^{inx}}{in})'$ *and do integration by parts.*

3 Compute the Laplace transforms of the following functions defined on $[0, \infty)$:

a) $f(t) = \sinh(At)$, A is a constant;

b) $f(t) = \cosh(At)$, A is a constant;

c)

$$f(t) = \begin{cases} 0 & \text{if } 0 < t < \pi \\ 1 & \text{if } t \geq \pi; \end{cases}$$

d)

$$f(t) = \begin{cases} 0 & \text{if } 0 < t < \pi \\ \cos t & \text{if } t \geq \pi; \end{cases}$$

e) $f(t) = t^2 e^t$;

f) $f(t) = e^t \cos t$;

g) $f(t) = e^t \sin t$.

4 Solve the following initial value problems using Laplace transforms:

a)

$$\begin{cases} y'' - 2y' + 2y = 6e^{-t} \\ y(0) = 0, y'(0) = 1; \end{cases}$$

Hint: You may use partical fraction decomposition of the following form: find A, B, C such that

$$\frac{As + B}{s^2 - 2s + 2} + \frac{C}{s + 1} = \frac{s + 7}{(s^2 - 2s + 2)(s + 1)},$$

then use Exercise [3] f), g).

b)

$$\begin{cases} y'' + y = f(t) \\ y(0) = y'(0) = 0, \end{cases}$$

where

$$f(t) = \begin{cases} 0 & \text{if } 0 < t < \pi \\ 1 & \text{if } t \geq \pi. \end{cases}$$

Hint: Find A, B, C such that

$$\frac{As + B}{s^2 + 1} + \frac{C}{s} = \frac{1}{(s^2 + 1)s},$$

then use Exercise [3] c), d).

Remark: In the first lecture, we mentioned how to find the inverse Laplace transform of a polynomial, but in real applications this can never happen, in fact the Laplace transform

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

will go to zero when s goes to infinity. Thus if F is a polynomial of s then $F \equiv 0$.