TMA4135
Matematikk 4D
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and Technology
Department of Mathematical
Sciences

Exercise set 1

1 Compute the Laplace transforms of the following functions:
   a) $f(t) = \sinh(At)$, A is a constant;

   b) $f(t) = \cosh(At)$, A is a constant;

   c) $f(t) = 0$ if $0 < t < \pi$; $f(t) = 1$ otherwise;

   d) $f(t) = 0$ if $0 < t < \pi$; $f(t) = \cos t$ otherwise;

   e) $f(t) = t^2 e^t$;

   f) $f(t) = e^t \cos t$;

   g) $f(t) = e^t \sin t$;

2 Solve the following initial value problems using Laplace transforms:
   a) $y'' - 2y' + 2y = 6e^{-t}$; $y(0) = 0, y'(0) = 1$;
   Hint: Find A, B, C such that
   $$\frac{As + B}{s^2 - 2s + 2} + \frac{C}{s + 1} = \frac{s + 7}{(s^2 - 2s + 2)(s + 1)},$$
   then use Exercise [1] f), g).

   b) $y'' + y = f(t)$, $f(t) = 0$ if $0 < t < \pi$; $f(t) = 1$ otherwise; $y(0) = 0$, $y'(0) = 0$;
   Hint: Find A, B, C such that
   $$\frac{As + B}{s^2 + 1} + \frac{C}{s} = \frac{1}{(s^2 + 1)s},$$
   then use Exercise [1] c), d).

3 By definition, a function f has period T if $f(t + T) = f(t)$. Show that the Laplace transform of a function f with period $T > 0$ is given by
   $$\mathcal{L}(f)(s) = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt.$$

4 ($\star\star$) Compare the Laplace transforms of the functions $h_n(t) = t^n$, $n = 1, 2, 3, \ldots$, using the general property $\mathcal{L}(h_m f)(s) = (-1)^m \frac{d^m \mathcal{L}(f)}{ds^m}(s)$. What can be said about the Laplace transform $\mathcal{L}(\frac{f}{h_1})(s)$?