Norwegian University of Science
and Technology
Department of Mathematical
Sciences

TMA4135
Matematikk 4D
Fall 2017

Exercise set 7

1 **a)** Let $f(t)$ be a piecewise continuous periodic function of period p. Show that its Laplace transform is given by

$$\mathscr{L}(f) = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t)\, \mathrm{d}t,$$

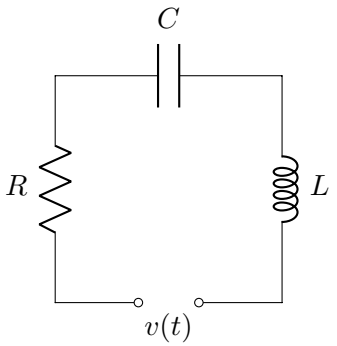
for $s > 0$. (**Hint**: Decompose $\int_0^\infty = \int_0^p + \int_p^{2p} + \cdots$).

b) Use this formula to compute the Laplace transform of the saw-tooth wave of period $p = 1$, given by

$$f(t) = t \quad \text{for } 0 \le t < 1.$$

2 Using the Laplace transform, find the current $i(t)$ in the circuit below, assuming zero initial current and charge, $R = 4\ \Omega$, $L = 1$ H, $C = 0.05$ F and

$$v(t) = \begin{cases} 34e^{-t}\ \text{V} & \text{if } 0<t<4, \\ 0\ \text{V} & \text{otherwise.} \end{cases}$$



3 Use the Laplace transform to solve the following ODEs.

a) $y'' + 4y' + 5y = \delta(t-1), \quad y(0) = 0,\ y'(0) = 3.$

b) $y'' + 5y' + 6y = \delta(t - \frac{1}{2}\pi) + u(t-\pi)\cos t, \quad y(0) = y'(0) = 0.$