The road from conditional Monte Carlo to improper priors and fiducial inference

Or trying to prove a false statement ...

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Abstract

We started out in 1997 on a project to clarify certain aspects of a conditional Monte Carlo method suggested by Engen and Lillegård. The method produces conditional samples given a sufficient statistic, and can hence be used to construct optimal inference procedures. In some cases it can also be used to eliminate nuisance parameters by conditioning. The steps in the algorithm give strong links to fiducial inference, and also to the foundations of statistics both in mathematical and philosophical terms. The talk will give an informal sketch of the initial method, and the many related foundational and practical issues.

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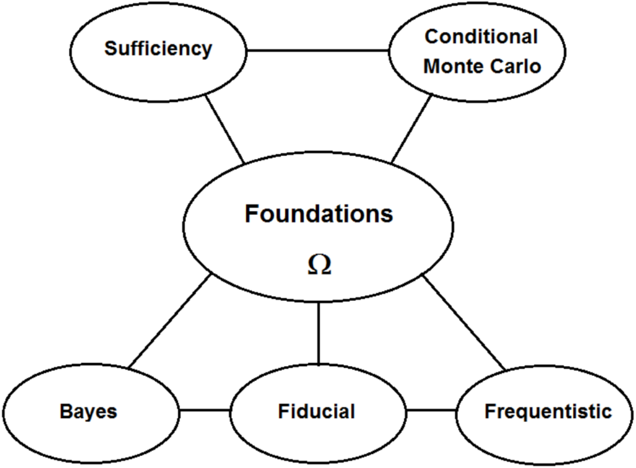
The Large Picture



From Conditional Monte Carlo to ...

- **Initial problem: Generate data X conditionally given a sufficient statistic T.**
- Tentative solution: Adjust parameter value θ for simulated data x so that the sufficient statistic is kept fixed equal to t.
- Corrected solution: The simulated data must be weighted, and the weight depends on an arbitrarily chosen improper distribution for the parameter.
- Realization after many years: Oh ... the resulting unweighted adjusted parameters are fiducial and the weighted are Bayes posterior ...
- Ingredients: Sufficiency, Optimal inference, Improper distributions, Fiducial versus Bayesian distribution, Borel paradox, Marginalization paradox, Axioms for statistics.

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Sufficiency with impropers

- **T is sufficient relatively to X for the parameter θ if $X \,|(T = t, \Theta = \theta)$ does not depend on θ.**
- Equivalently: X, Θ are conditionally independent given T, so $(\Theta \,|\, X, T) \sim (\Theta \,|\, T)$. If, additionally, $T = \tau(X)$, then $(\Theta \,|\, X) \sim (\Theta \,|\, T) \sim$ Bayes posterior.
- Theorem: Sufficiency implies that $[X \,|(T = t, \Theta = \theta)] \sim [X \,|\, T = t]$.
- Proof in discrete case: $E^\theta(\phi(X) \,|\, T = t) =$

$$\frac{E^\theta(\phi(X)(T = t))}{E^\theta(T = t)} = \frac{\int \pi(\theta) E^\theta(\phi(X)(T = t))\, d\theta}{\int \pi(\theta) E^\theta(T = t)\, d\theta}$$

- Beware: $X \,|\, Y = y$ does not depend on y does NOT imply that $(X \,|\, Y) \sim X$.

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Sufficiency and optimal inference

- **Sufficiency principle: Valid inference must be based on T for all sufficient T. It can be disputed. Halmos argument relies on accepting randomized procedures.**
- If $\phi(X)$ is an estimator, then $E^\theta(\phi(X) \mid T)$ is an estimator with smaller (or equal) convex risk. If T is complete (and minimal), then it is the unique optimal estimator.
- An exact test for $(H_0 : \alpha = \alpha_0, \theta$ arbitrary) is obtainable if T is sufficient for the nuisance parameter θ. This gives exact confidence distributions. Lehmann: Optimality follows from this with additional assumptions.
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An erroneous (but nice) argument

- **A joint fiducial model $X = \chi(U, \Theta)$ and $T = \tau(U, \Theta)$.**
- Assume $\tau(u, \hat{\theta}(u, t)) = t$ and $\hat{\theta}(u, \tau(u, \theta)) = \theta$.
- For any such model one may calculate (formally):
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- What is wrong?
- Answer: There is a Borel paradox type error in the argument. The event $(\tau(U, \theta) = t) = (\hat{\theta}(U, t) = \theta)$ has zero probability. The σ-fields must be equal.

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Three correct arguments

- **Pivotal argument: The previous calculation is correct by assuming an additional pivotal structure.**
- Discrete case argument: Elementary calculation possible, and added weight arises from added law for the parameter. A limit argument gives the continuous case.
- Sampling argument: Everything is reduced to the problem of sampling from $(U, \Theta) \,|\, T = t$ which can be done in two steps:
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The pivotal argument

- **Assume that $\tau(u, \theta) = \theta \circ r(u)$ where $\circ$ defines a quasi-group (or loop) multiplication, and $\Omega_\Theta = \Omega_T = \Omega_R$ have been identified.**
- The previous calculation is then correct since $[\tau(U, \theta) = t] = [\theta \circ R = t] = [R = \theta^{-1} \circ t] = [t \circ R^{-1} = \theta] = [\hat{\theta}(U, t) = \theta]$, and all involved functions are one-one.
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The sampling argument

- **Theorem:** $(U \mid T = t) \sim w_t(u)\, \mathbf{P}_U(du)$ **when** $\tau(u, \Theta) \sim w_t(u)\mu(dt)$ **is assumed.**
- Proof from change-of-variables theorem: $E(\phi(U, T)) =$

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- Conclusion: Sampling u from $U \mid T = t$ is obtained by sampling u from U with weight $w_t(u)$.
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- Frequentist: Uncertainty indirectly from properties of the instrument in use.
- Fiducial: Interpretation unproblematic when identified with a special case of frequentist inference = Confidence distributions.
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Bayes versus Frequentist: No conflict

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Final comments on the involved ideas

And some possibly often forgotten

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- Much work remains regarding general theory and good examples.

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