Plenumsregning #12

Problem 1.

Use D'Alembert's method to find the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

for $t > 0$ and all $x \in \mathbb{R}$ with initial conditions

$$u(x,0) = \sin x \qquad \text{and} \qquad \frac{\partial u(x,0)}{\partial t} = e^x.$$

Problem 2. (Damped wave equation)

One way to include dissipation in wave modelling is via linear damping. In that case, the wave equation becomes

$$\frac{\partial^2 u}{\partial t^2} + 2\gamma \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2},$$

in which c is the wave speed and $\gamma > 0$ is the damping parameter. As for the standard wave equation, we need boundary and initial conditions. Considering the following ones

$$\begin{aligned} u(0,t) &= 0\,, \\ u(L,t) &= 0\,, \\ u(x,0) &= f(x)\,, \\ \left.\frac{\partial u}{\partial t}\right|_{t=0} &= 0 \end{aligned}$$

and assuming $\gamma < c\pi/L$, use the separation of variables technique to show that the solution to this problem is

$$u(x,t) = \sum_{n=1}^{\infty} C_n e^{-\gamma t} \sin(\lambda_n x) \cos\left(t\sqrt{(c\lambda_n)^2 - \gamma^2}\right), \quad \text{with } \lambda_n := \frac{n\pi}{L}, \; n \in \mathbb{N},$$

and determine the constants C_n in terms of $f(x)$.

Resten av av plenumsregningen er bare relevant for 4N

Problem 3. (Solving PDEs using Fourier Transform)

Oppgave 5 fra eksamen høst 2022

Hvis tid (Ikke relatert til denne ukas pensum, men for å komme igang med neste ukes plan om å regne fra eksamen 2022.):

Problem 4.

Oppgave 9 fra eksamen høst 2022.