



Kunnskap for en bedre verden

TMA4130 MATEMATIKK 4N

Lecture 8: Numerical Methods for Nonlinear Equations

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Important!

Remember to use **check-in** for this room!

Outline for the Day

- ▶ Newton's Method
- ▶ Systems of Nonlinear Equations

Short Summary

- ▶ Finding a root of a nonlinear equation $f(x) = 0$:
Newton's Method:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$

An initial value x_0 and two function evaluations (f and f') are needed.

Short Summary

- ▶ Systems of Nonlinear Equations: Finding a root of a **system** of nonlinear equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$:

$$\mathbf{x}_{k+1} = \mathbf{x}_k - J(\mathbf{x}_k)^{-1}\mathbf{f}(\mathbf{x}_k).$$

In practice: Jacobian linear system

$$J(\mathbf{x}_k)\mathbf{h}_k = -\mathbf{f}(\mathbf{x}_k)$$

via Gaussian elimination. Next iterate:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{h}_k.$$

Resources

- ▶ Lecture notes on numerical solutions of nonlinear equations by Morten Nome (in Norwegian): <https://www.math.ntnu.no/emner/TMA4125/2019v/notater/06-likningslosere.pdf>
- ▶ Lecture notes on numerical solutions of nonlinear equations by Anne Kværnø: https://wiki.math.ntnu.no/_media/tma4130/2020h/nonlinearequations.pdf
- ▶ Book *Numerical Mathematics and Computing* by Ward Cheney and David Kincaid, 7th edition, 2013, Brooks/Cole.

Next lecture

- ▶ Ordinary Differential Equations
- ▶ Laplace Transform