

TMA4130 MATEMATIKK 4N PROBLEM SHEET: 3RD WEEK

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1.

- (i) Let $r(t)$ be a bounded continuous function that is zero outside of $[-M, M]$, $M > 0$. Express the solution to the following problem as an integral:

$$y'' + 2y' - 8y = r, \quad y(0) = -y'(0) = a.$$

- (ii) Find the inverse Laplace transform of $F(s) = \log(s/(s-1))$.

2.

- (i) Use the Laplace transform to find the general solution to the following system of ODEs:

$$\begin{aligned} y_1' &= y_1 + y_2 \\ y_2' &= -5y_1 - 3y_2. \end{aligned}$$

- (ii) With respect to (i), show that if $(y_1(0), y_2(0)) = (0, 0)$, then $(y_1(t), y_2(t)) = (0, 0)$. What happens to $(y_1(t), y_2(t))$ as $t \rightarrow \infty$ if $(y_1(0), y_2(0)) \neq (0, 0)$?

- (iii) Use the Laplace transform to find the general solution to the following system of ODEs:

$$\begin{aligned} y_1' &= 4y_1 - 2y_2 \\ y_2' &= 3y_1 - y_2. \end{aligned}$$

- (iv) With respect to (iii), show that if $(y_1(0), y_2(0)) = (0, 0)$, then $(y_1(t), y_2(t)) = (0, 0)$. What happens to $(y_1(t), y_2(t))$ as $t \rightarrow \infty$ if $(y_1(0), y_2(0)) \neq (0, 0)$?

3. Compute the Fourier coefficients associated with the following functions:

- (i) $f(t) = \frac{t}{2\pi} - \left\lfloor \frac{t}{2\pi} \right\rfloor$, where, again, $[t]$ is the largest integer smaller than t ,
 (ii) $f(t) = \sum_{n \in \mathbb{Z}} g(t - 2n\pi)$, where $g(x) = |x| \mathbb{1}_{[-\pi, \pi)}(x)$.

4.

- (i) Compute the Fourier coefficients associated with $f(x) = \sum_{n \in \mathbb{Z}} g(t - 2n\pi)$, where $g(x) = x^2 \mathbb{1}_{[-\pi, \pi)}(x)$.
 (ii) Use the convergence theorem (Cap. 11.1, Theorem 2 *Representation by a Fourier Series*) and show that

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}.$$