Norwegian University of Science and Technology
Department of Mathematical Sciences

# TMA4130 Matematikk 4N Fall 2018

**Exercise set 8**

The theory and the codes are taken from the Jupyter notebook on *Polynomial Interpolations.*

Exercises supposed done by hand are marked with (H), exercises in which you are supposed to use/modify code in the Jupyter notebook are marked with an (J). For Jupyter-exercises, hand in a screen-dump of the relevant cell with output.

**1** Consider the data points

| $x_i$ | $-1$ | $-1/2$ | $1/2$ | $1$ |
|---|---|---|---|---|
| $f(x_i)$ | $-1/2$ | $-5/4$ | $1/4$ | $5/2$ |

**a)** (H) Use Lagrange interpolatation to find the polynomial of minimal degree interpolating these points. Express the polynomial in the form

$$p_n(x) = a_n x^n + \cdots + a_1 x + a_0.$$

Use this to find an approximation to $f(0)$.

**b)** (J) Plot the polynomial you found on the interval $[-1.5, 1.4]$.
Include a plot of the interpolation points, and confirm by this that the polynomial really is the interpolation polynomial.

**c)** (J, no hand-in) Plot the interpolation polynomial again, but this time by the use of the python functions `cardinal` and `lagrange`, and make sure that this plot is exactly the same as in the previous point.

By doing the last two points, you confirm that you have found the correct interpolation polynomial, and that you know how to use the functions `cardinal` and `lagrange`.

**2** (J) The population of Norway in the period from 1993 to 2018 is, according to SSB,

| year | 1993 | 1998 | 2003 | 2008 | 2013 | 2018 |
|---|---|---|---|---|---|---|
| population | 4299167 | 4417599 | 4552252 | 4737171 | 5051275 | 5295619 |

Use the interpolation polynomial, through the functions `kardinal` and `lagrange`, to estimate the population in the years 2000 and 2010. Predict the population in 2025 and 2030. Comment on the results.

(The population in 2000 was 4478497, in 2010 it was 4858199).

**3** Consider the function $f(x) = x^2 \cos x$.

**a)** (H) Find the polynomial of degree 3 that interpolates $f(x)$ at four

- equally distributed nodes
- Chebyshev points

in the interval $[-1, 2]$.

**b)** (H) Find by hand a bound for the maximal interpolation error in that interval in these two cases.

**c)** (J) Confirm your results numerically:
Plot the function and the interpolation polynomial, measure the maximal interpolation error and compare it with your theoretical result.

**d)** (J) Repeat the experiments numerically, with $n + 1$ interpolation points, using $n = 5, 10, 15, 20$.
In this case, we are only interested in the measured maximum error and the error bounds.

**Hint:**

$$\frac{d^n}{dx^n} x^2 \cos(x) = \begin{cases} (-1)^{n/2}\big(x^2 \cos(x) + 2nx \sin(x) - n(n-1)\cos(x)\big) & \text{for } n \text{ even,} \\ (-1)^{(n+1)/2}\big(x^2 \sin(x) - 2nx \cos(x) - n(n-1)\sin(x)\big) & \text{for } n \text{ odd.} \end{cases}$$