Formula Sheet. TMA4125 Matematikk 4N, Spring 2025.

Fourier Transform. The Fourier Transform $\hat{f} = \mathcal{F}(f)$ and its inverse $f = \mathcal{F}^{-1}(\hat{f})$ are

$$\hat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \mathrm{e}^{-\mathrm{i}\omega x} \, \mathrm{d}x \quad \text{and} \quad f(x) = \mathcal{F}^{-1}(\hat{f})(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) \mathrm{e}^{\mathrm{i}\omega x} \, \mathrm{d}\omega$$

Laplace Transform. The Laplace transform $F(s)$ of $f(t)$, $t \geq 0$, reads

$$F(s) = \int_0^{\infty} \mathrm{e}^{-st} f(t) \, \mathrm{d}t$$

List of Fourier Transforms.

$f(x)$	$\hat{f}(\omega)$
e^{-ax^2}	$\frac{1}{\sqrt{2a}} \mathrm{e}^{-\frac{\omega^2}{4a}}$
$\mathrm{e}^{-a\lvert x\rvert}$	$\sqrt{\frac{2}{\pi}} \frac{a}{\omega^2 + a^2}$
$\frac{1}{x^2 + a^2}$ for $a > 0$	$\sqrt{\frac{\pi}{2}} \frac{\mathrm{e}^{-a\lvert\omega\rvert}}{a}$
$\begin{cases} 1 & \text{for } \lvert x\rvert < a \\ 0 & \text{otherwise} \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin(\omega a)}{\omega}$
$\sqrt{\frac{2}{\pi}} \frac{\sin(ax)}{x}$	$\begin{cases} 1 & \text{for } \lvert\omega\rvert < a \\ 0 & \text{otherwise} \end{cases}$

List of Laplace Transforms.

$f(t)$	$F(s)$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
$\cosh(\omega t)$	$\frac{s}{s^2 - \omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{s^2 - \omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s - a}$
$f(t-a)u(t-a)$	$\mathrm{e}^{-sa} F(s)$
$\delta(t-a)$	e^{-sa}

Trigonometric identities.

- $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$
- $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$
- $\cos(2\alpha) = 2\cos^2(\alpha) - 1 = 1 - 2\sin^2(\alpha)$
- $\mathrm{e}^{\mathrm{i}\alpha} = \cos\alpha + \mathrm{i}\sin\alpha$
- $2\sin\alpha\cos\beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
- $2\cos\alpha\sin\beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$
- $2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$
- $2\sin\alpha\sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$

Integrals.

- $\int x^n \cos ax \mathrm{d}x = \frac{1}{a} x^n \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \mathrm{d}x$
- $\int x^n \sin ax \mathrm{d}x = -\frac{1}{a} x^n \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \mathrm{d}x$

Order conditions for Runge-Kutta methods.

p	Conditions	p	Conditions
1	$\sum_{i=1}^{s} b_i = 1$	4	$\sum_{i=1}^{s} b_i {c_i}^3 = \frac{1}{4}$
2	$\sum_{i=1}^{s} b_i c_i = \frac{1}{2}$		$\sum_{i=1}^{s} \sum_{j=1}^{s} b_i c_i a_{ij} c_j = \frac{1}{8}$
3	$\sum_{i=1}^{s} b_i {c_i}^2 = \frac{1}{3}$		$\sum_{i=1}^{s} \sum_{j=1}^{s} b_i a_{ij} {c_j}^2 = \frac{1}{12}$
	$\sum_{i=1}^{s} \sum_{j=1}^{s} b_i a_{ij} c_j = \frac{1}{6}$		$\sum_{i=1}^{s} \sum_{j=1}^{s} \sum_{k=1}^{s} b_i a_{ij} a_{jk} c_k = \frac{1}{24}$