# Preliminaries

12.01.'23

1-18, We have $e_{k+1} = C e_k^p$ (don't know p)

$$e_{k+2} = C e_{k+1}^p$$

$$\Rightarrow \frac{e_{k+2}}{e_{k+1}} = \left(\frac{e_{k+1}}{e_k}\right)^p$$

$$\Rightarrow \log\left(\frac{e_{k+2}}{e_{k+1}}\right) = p \log\left(\frac{e_{k+1}}{e_k}\right)$$

$$\Rightarrow p \approx \frac{\log\left(\frac{e_{k+2}}{e_{k+1}}\right)}{\log\left(\frac{e_{k+1}}{e_k}\right)}$$

## A Boundary Value Problem

13.01.'23

1-24, We start with $u(x,t)$ the concentration of A at $x$ at time $t \geq 0$.

Fix some $x = x(t)$ that "follows a set of atoms"

$$\frac{d}{dt} u = \frac{d}{dt} u(x(t), t)$$

We have

(1) $$\frac{d}{dt} u = \alpha \frac{\partial u^2}{\partial x^2} - k u^n$$

diffusion of A (Fick's law)

reaction of $nA \to B + C$ $(n=1, n=2)$

(2) By chain rule

convexion $v(x) = V$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial x}{\partial t} \cdot \frac{\partial u}{\partial x} \Rightarrow \frac{\partial u}{\partial t} = \frac{du}{dt} - V \frac{\partial u}{\partial x}$$

(3) with steady state

$$0 = \frac{\partial u}{\partial t} = \alpha \frac{\partial u^2}{\partial x^2}$$

③ steady state

$$0 = \frac{\partial u}{\partial t} = \alpha \frac{\partial u^2}{\partial x^2} - v \frac{\partial u}{\partial x} - k u^4$$

Since $u(x,t)$ does not change in time: $u(x)$

$$\Rightarrow 0 = \alpha u'' - v u' - k u^4$$

1-28/

$f(x+h) = f(x) + f'(x)h + \frac{1}{2} f''(\xi) h^2$

Taylor!

$$e(x,h) = f'(x) - \frac{f(x+h) - f(x)}{h}$$

$$= \frac{f'(x)h}{\cancel{h}} - \frac{\cancel{f(x)} + \boxed{f'(x)h} + \frac{1}{2}f''(\xi)h^2 - \cancel{f(x)}}{\boxed{h}}$$

$$= \frac{1}{2} f''(\xi) h, \qquad \xi \in (x, x+h)$$

$$= O(h)$$

1-30/

$$U_0 = u_a$$

$$\frac{U_2 - 2U_1 + U_0}{h^2} + \frac{U_2 - U_0}{2h} p(x_1) + q(x_1) U_1 = r(x_1)$$

$$\frac{U_{i+1} - 2U_i + U_{i-1}}{h^2} + \frac{U_{i+1} - U_{i-1}}{2h} p(x_i) + q(x_i) U_i = r(x_i)$$

$(i = 1, \dots, N-1)$

$$\left(\frac{1}{h^2} - \frac{1}{2h} p(x_i)\right) U_{i-1} + \left(-\frac{2}{h^2} + q(x_i)\right) U_i + \left(\frac{1}{h^2} + \frac{1}{2h} p(x_i)\right) U_{i+1} = r(x_i)$$

multiply with $h^2$

$$\left(1 - \frac{h}{2} p(x_i)\right) U_{i-1} + \left(-2 + h^2 q(x_i)\right) U_i + \left(1 + \frac{h}{2} p(x_i)\right) U_{i+1} = h^2 r(x_i)$$

$$U_N = u_b$$