NB! Kan bare brukes i tillegg til Jupyternotater.

Og på eget ansvar!

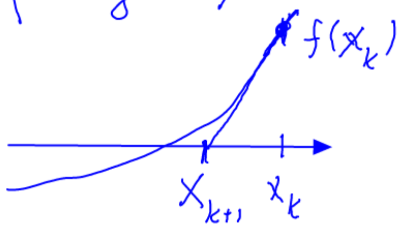
Skalare ligninger: $f(x) = 0$

Newton : $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

Eks: $x^3 - y + 1/4 = 0$

$x^2 + y^2 - 1 = 0$ $\Rightarrow$

Gitt $\vec{f}(\vec{x}) = 0$ $\quad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \vec{f}(\vec{x}) = \begin{pmatrix} x^3 - y + 1/4 \\ x^2 + y - 1 \end{pmatrix}$



$$f(x_k) + f'(x_k)\cdot(x_{k+1} - x_k) = 0$$

Newton's metode i 2 dimensjoner.

$\vec{f}(\vec{x}) = 0 \Rightarrow$

$$f(x,y) = 0$$
$$g(x,y) = 0$$

$\vec{f} = \begin{pmatrix} f \\ g \end{pmatrix}$

$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

Anta at denne har en løsning $\vec{r} = (r_x, r_y)^T$.

Anta at vi kjenner $\vec{x}_k = (x_k, y_k)^T \approx \vec{r}$

$x_k = r_x - \Delta x, \quad y_k = r_y - \Delta y$

Taylorutvikler $\vec{f}$ rundt $\vec{x}_k$ (som er kjent).

$f(r_x, r_y) = 0 \quad =$

$g(r_x, r_y) = 0$

alt beregnes i (x_k, y_k)

$$0 = f(x_k + \Delta x, y_k + \Delta y) \overset{\text{Taylor}}{=} f(x_k, y_k) + \frac{\partial f}{\partial x}\cdot \Delta x_k + \frac{\partial f}{\partial y}\cdot \Delta y_k + \cancel{h.o.l.}$$

$$0 = g(x_k + \Delta x, y_k + \Delta y) = g(x_k, y_k) + \frac{\partial g}{\partial x}\cdot \Delta x_k + \frac{\partial g}{\partial y}\cdot \Delta y_k + \cancel{h.o.l.}$$

Bedre løsning $x_{k+1} = x_k + \Delta x_k, \quad y_{k+1} = y_k + \Delta y_k$

h.o.l. høyere ordens ledd: $\mathcal{O}(\Delta x^2 + \Delta x \cdot y + \Delta y^2)$

Jakobimatrise (Jakobian)

$$J(x,y) = \begin{pmatrix} \frac{\partial f}{\partial x}(x,y) & \frac{\partial f}{\partial y}(x,y) \\ \frac{\partial g}{\partial x}(x,y) & \frac{\partial g}{\partial y}(x,y) \end{pmatrix}$$

Newtons metode for systemer:

$$\begin{pmatrix} f(x_k, y_k) \\ g(x_k, y_k) \end{pmatrix} + J(x_k, y_k) \cdot \Delta\vec{X}_k = 0$$

Løs et lineært lign. syst.

$$\vec{X}_{k+1} = \vec{X}_k + \Delta\vec{X}_k \quad , k = 0, 1, 2, \dots$$

Må starte med en $\vec{X}_0 = (x_0, y_0)$.

Eks:
$$f = x^3 - y + 1/4$$
$$g = x^2 + y^2 - 1$$

$$J(x,y) = \begin{pmatrix} 3x^2 & -1 \\ 2x & 2y \end{pmatrix}$$

Kvadratisk konvergens: $\vec{e}_k = \vec{r} - x_k$

$$\|e_{k+1}\| \le M \cdot \|e_k\|^2$$

$$\|e_k\| = \begin{cases} \max |e_{i,k}| \\ \sqrt{\sum_{i=1}^{n} e_{i,k}^2} \end{cases}$$

$$\vec{x} = (x, y) \overset{python}{\Longrightarrow} [\overset{x}{x[0]}, \overset{y}{x[1]}]$$

$$\vec{f}(x,y) = \begin{pmatrix} xe^{y} - 1 \\ -x^2 + y - 1 \end{pmatrix} \qquad \vec{x}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\vec{f}(0,0) = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$J = \begin{pmatrix} e^{y} & x \cdot e^{y} \\ -2x & 1 \end{pmatrix}, \qquad J(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

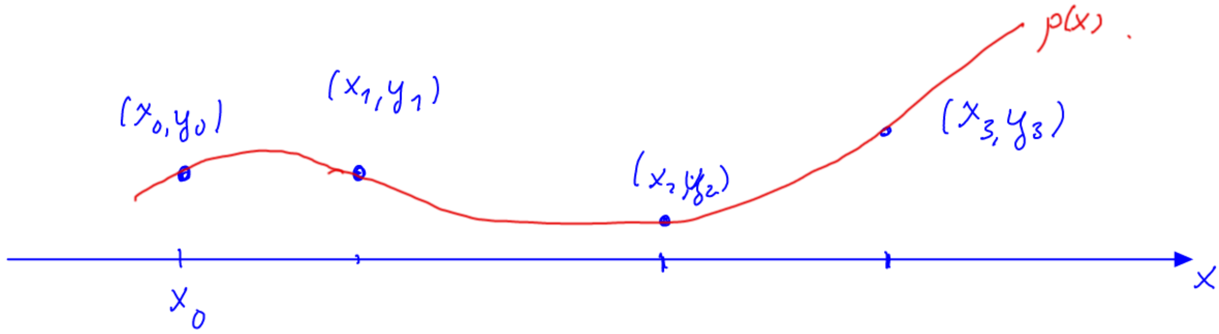
1. skritt i Newton.

Løs $\quad \vec{f}(0,0) + J(0,0)\,\Delta\vec{x}_0 = \vec{0} \quad$ mhp. Δx_0

$$\begin{pmatrix} -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Delta\vec{x}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \Delta\vec{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \leftarrow$$

$$\vec{x}_1 = \vec{x}_0 + \Delta\vec{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



Gitt $n+1$ datapunkter $(x_i, y_i)_{i=0}^{n}$

med distinkte x_i (forskjellige). Noder.

Oppgave: Finn det polynomet av lavest mulig grad s.a.

$$p(x_i) = y_i \quad , \overbrace{i = 0, \cdots, n}^{n+1 \text{ bet.}}$$

Interpolasjons-betingelsen

interpolasjonspolynomet.

- Polynom av grad n:

$$p_n(x) = a_n x^n + a_{n-1} x^{n-1} \cdots \quad a_1 x + a_0 \quad , \quad a_i \in \mathbb{R}$$

Lavest mulig grad: n

$$x^2 - 2x + 1 \quad = (x-1)^2$$

Gitt $(x_i, y_i)_{i=0}^{n}$: Hvordan finne $p_n(x) \in \mathbb{P}_n$

Definer kardinalfunksjoner:

$$l_i(x) = \frac{(x-x_0)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_0)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)} \quad , \quad i = 0, \cdots, n$$

Eks. $x_0 = 1, \; x_1 = 2, \; x_2 = 4$

$$l_0 = \frac{(x-2)(x-4)}{(1-2)(1-4)} \in \mathbb{P}_2 \quad , \quad l_0(1) = 1 \quad , \quad l_0(2) = 0$$

Egenskaper: • $l_i(x) \in \mathbb{P}_n$

• $l_i(x_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

Gitt $(x_i, y_i)_{i=0}^{n}$

Interpolasjonspolynomet:

$$p_n(x) = \sum_{i=0}^{n} y_i \cdot l_i(x) = y_0 l_0(x) + \dots + y_n \cdot l_n(x)$$

Fordi $p_n \in \mathbb{P}_n$

$$p_n(x_j) = \sum_{i=0}^{n} y_i \, l_i(x_j) = y_j \cdot \overbrace{l_j(x_j)}^{=1} = y_j$$