

Formler i numerikk

- La $p(x)$ være et polynom av grad $\leq n$ som interpolerer $f(x)$ i punktene $x_i, i = 0, 1, \dots, n$. Forutsatt at x og alle nodene ligger i intervallet $[a, b]$, så gjelder

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{i=0}^n (x - x_i).$$

- Newtons dividerte differansers interpolasjonspolynom $p(x)$ av grad $\leq n$:

$$p(x) = f[x_0] + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] \\ + \dots + (x - x_0)(x - x_1) \dots (x - x_{n-1})f[x_0, \dots, x_n]$$

- Numerisk derivasjon:

$$f'(x) = \frac{1}{h} [f(x+h) - f(x)] + \frac{1}{2} h f''(\xi) \\ f'(x) = \frac{1}{2h} [f(x+h) - f(x-h)] - \frac{1}{6} h^2 f'''(\xi) \\ f''(x) = \frac{1}{h^2} [f(x+h) - 2f(x) + f(x-h)] - \frac{1}{12} h^2 f^{(4)}(\xi)$$

- Simpsons integrasjonsformel:

$$\int_{x_0}^{x_2} f(x) dx \approx \frac{h}{3} (f_0 + 4f_1 + f_2)$$

- Newtons metode for ligningssystemet $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ er gitt ved

$$J^{(k)} \cdot \Delta \mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)}) \\ \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}.$$

- Iterative teknikker for løsning av et lineært ligningssystem

$$\sum_{j=1}^n a_{ij} x_j = b_i, \quad i = 1, 2, \dots, n$$

$$\text{Jacobi: } x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right)$$

$$\text{Gauss-Seidel: } x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right)$$

- Heuns metode for løsning av $\mathbf{y}' = \mathbf{f}(x, \mathbf{y})$:

$$\mathbf{k}_1 = h\mathbf{f}(x_n, \mathbf{y}_n) \\ \mathbf{k}_2 = h\mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{k}_1) \\ \mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)$$

Tabell over noen Laplace-transformer

$f(t)$	$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n ($n = 0, 1, 2, \dots$)	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s - a}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$e^{at} \cos \omega t$	$\frac{s - a}{(s - a)^2 + \omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s - a)^2 + \omega^2}$

Tabell over noen Fourier-transformer

$f(x)$	$\hat{f}(w) = \mathcal{F}(f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx$
$g(x) = f(ax)$	$\hat{g}(w) = \frac{1}{a} \hat{f}\left(\frac{w}{a}\right)$
$u(x) - u(x - a)$	$\frac{1}{\sqrt{2\pi}} \left(\frac{\sin aw}{w} - i \frac{1 - \cos aw}{w} \right)$
$u(x) e^{-x}$	$\frac{1}{\sqrt{2\pi}} \left(\frac{1}{1 + w^2} - i \frac{w}{1 + w^2} \right)$
e^{-ax^2}	$\frac{1}{\sqrt{2a}} e^{-\frac{w^2}{4a}}$
$e^{-a x }$	$\sqrt{\frac{2}{\pi}} \frac{a}{w^2 + a^2}$