Norwegian University of Science
and Technology
Institute of mathematical sciences

Exercise set 9

1 The following table gives the total petroleum production on the Norwegian shelf in selected years. The t-column denotes years since 2000 (so $t = 1$ is 2001 etc.), The f-column is total petroleum production in millions of Sm^3 oil equivalents.

t_j	f_j
0	244
4	264
8	243
12	226

a) Find the Lagrange interpolation polynomial for these data.

b) Use the polynomial from **a)** to find an approximation to the petroleum production in 2010. For comparison, the actual production this year was 231 million Sm^3.

c) Use the polynomial to find an approximation for the oil production in 2020. Is this realistic? Using the polynomial, what is the approximation for the oil production in 1993?

2 **a)** In general, which polynomial degree do you need to interpolate 5 data points?

b) Find the interpolation polynomial for the data $(-4, 50)$, $(-2, 18)$, $(0, 2)$, $(2, 2)$, $(4, 18)$ by using a divided difference table. What is the degree of this polynomial? How can you see the degree from the divided difference table?

3 In this problem, we want to estimate the integral $I = \int_0^2 \cos x^2 \, \mathrm{d}x$ by using different techniques.

a) Approximate I using the rectangular rule on 4 subintervals of equal length.

b) Approximate I using the trapezoidal rule on 4 subintervals of equal length.

c) Approximate I using Simpson's rule on 4 subintervals of equal length.

4 In this problem, we want to establish expressions for the error in some common numeric differentiation formulas to approximate f' and f'' In both subproblems, we assume that f is an analytic function, that is, f is equal to its Taylor expansion around the point x.

$$f(x+t) = f(x) + f'(x)t + \frac{f''(x)}{2!}t^2 + \cdots + \frac{f^{(n)}}{n!}t^n + \cdots$$

In the following h is a small parameter, so the dominant term of an expansion in h is the term with the smallest exponent.

a) The *forward difference* formula for f' says

$$f'(x) \approx \Delta f(x) = \frac{f(x+h) - f(x)}{h}$$

Find the dominant term of the error $\Delta f(x) - f'(x)$ using the Taylor expansion of f.

b) [1] The *central difference* formula for f' says

$$f'(x) \approx \delta_h f(x) = \frac{f(x+h) - f(x-h)}{2h}$$

Find the dominant term of the error $\delta_h f(x) - f'(x)$ using the Taylor expansion of f.

c) Find the dominant term of the error

$$\delta_{h/2}^2 f(x) - f'' = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} - f''.$$

[1] The notation of Kreyszig Sec 19.5. is generalized, Kreyszig's δ corresponds to $\delta_{h/2}$ in the notation here.