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Department of Mathematical
Sciences

TMA4122
Matematikk 4M &
TMA4130
Matematikk 4N
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Exercise set 4

In all problems you are supposed to show the details of your work and describe what you are doing.

- 1 Use D'Alembert's method to find the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

for $t > 0$ and all $x \in \mathbb{R}$ with initial conditions

$$u(x, 0) = \cos x \quad \text{and} \quad \frac{\partial u(x, 0)}{\partial t} = xe^{-x^2}.$$

- 2 The PDE

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - \mu \frac{\partial u}{\partial t}$$

with constants $c > 0$ and $\mu > 0$ can be used for modeling the oscillations of a string with damping. We assume in the following that $c = 1$ and $\mu = 2$, and that the string has a length of π and satisfies the boundary conditions

$$u(0, t) = 0 \quad \text{and} \quad u(\pi, t) = 0.$$

- a) Find all the solutions u of this equation with the given boundary conditions that are of the form $u(x, t) = F(x)G(t)$.
- b) Find the solution of this equation with initial conditions

$$u(x, 0) = x(\pi - x) \quad \text{and} \quad \frac{\partial u(x, 0)}{\partial t} = 0.$$

- c) Verify that the function

$$u(x, t) = te^{-t} \sin(x)$$

solves the PDE with the given boundary conditions and the initial conditions

$$u(x, 0) = 0 \quad \text{and} \quad \frac{\partial u(x, 0)}{\partial t} = \sin(x).$$

Check whether you have also found this solution in part a) of this exercise.¹

¹If you have not, check possible exceptional cases in your solutions or/and revise the chapter on ordinary differential equations from your Mathematics 3 course.

- 3 We consider a long, thin gold bar of length $L = 1\text{m}$, one end of which is held at a constant temperature of 0°C whereas the other end is insulated. This can be modeled by the heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

with boundary conditions

$$u(0, t) = 0 \quad \text{and} \quad \frac{\partial u(1, t)}{\partial x} = 0.$$

Here $c^2 \approx 1.27 \times 10^{-4} \text{m}^2/\text{s}$ is the thermal diffusivity of gold.

- a) Compute all solutions of this equation with the given boundary conditions that are of the form $u(x, t) = F(x)G(t)$.
- b) Assume that the initial temperature distribution in the gold bar is given by

$$u(x, 0) = 100 \sin(\pi x/2).$$

(u is given in degree Celsius). How long does it take, until the insulated end of the gold bar has cooled down to 20°C ?