Norwegian University of Science and Technology Department of Mathematical Sciences	TMA4130 Matematikk 4N Fall 2014 **Exercise set 10**

In all problems you are supposed to show the details of your work and describe what you are doing.

Kreyszig, Chap. 19.3

1 Consider the data points

x_i	0	1	2	4	6
$f(x_i)$	1	9	23	93	259

a) Use Newton interpolation and divided differences in order to find the interpolation polynomial of minimal degree that interpolates these points.

b) Use the interpolation polynomial in order to obtain an estimate of f at $x = 3$.

Kreyszig, Chap. 19.5

2 Use the rectangular rule and the trapezoidal rule with $n = 4$ in order to approximate the integral

$$J = \int_0^1 e^{2x} \sin(3x)\, dx.$$

3 We want to approximate the integral

$$J = \int_0^1 \sin(\exp(x^2))\, dx.$$

a) Compute two approximations using the trapezoidal rule with $n = 4$ and $n = 8$.

b) Use your numerical results in order to estimate *numerically* the approximation error for $n = 8$.
(Apply the formula on error estimation by halving h from the lecture or Kreyszig's book.)

$\boxed{4}$ We want to approximate the integral

$$J = \int_1^2 \frac{1}{x}\, dx.$$

a) Use the trapezoidal rule with $n = 4$ for approximating J.

b) Use Simpson's rule with $2m = 4$ for approximating J.

c) Derive exact upper bounds for the approximation errors (for both the trapezoidal rule and Simpson's rule) using the formulas from the lecture (or Kreyszig's book).

d) How large should one choose n (for the trapezoidal method) or m (for Simpson's method) in order to guarantee that the approximation error is smaller than 10^{-6}?