

Fourier

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega x} d\omega$	$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$
$f * g(x)$	$\sqrt{2\pi} \hat{f}(\omega) \hat{g}(\omega)$
$f'(x)$	$i\omega \hat{f}(\omega)$
e^{-ax^2}	$\frac{1}{\sqrt{2a}} e^{-\omega^2/4a}$
$e^{-a x }$	$\sqrt{\frac{2}{\pi}} \frac{a}{\omega^2 + a^2}$
$\frac{1}{1+x^2}$	$\sqrt{\frac{\pi}{2}} e^{- \omega }$
$f(x) = 1$ for $ x < a$, 0 otherwise	$\sqrt{\frac{2}{\pi}} \frac{\sin \omega a}{\omega}$

Laplace transform

$f(t)$	$F(s) = \int_0^{\infty} e^{-st} f(t) dt$
$f'(t)$	$sF(s) - f(0)$
$tf(t)$	$-F'(s)$
$e^{at} f(t)$	$F(s - a)$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s - a}$
$f(t - a)u(t - a)$	$e^{-sa} F(s)$
$\delta(t - a)$	e^{-as}

Numerics

- Newton's method: $x_{k+1} = x_k - f(x_k)/f'(x_k)$.
- Newton's method for systems: $\mathbf{J}^{(k)}(\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}) = -\mathbf{f}(\mathbf{x}^{(k)})$ with $(\mathbf{J}^{(k)})_{ij} = \partial_j f_i^{(k)}$
- Lagrange interpolation polynomial: $L_k(x) = \frac{(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$,
 $p_n(x) = \sum_{k=0}^n L_k(x)f(x_k)$
- Trapezoid rule: $\int_a^b f(x) dx \approx h \left[\frac{1}{2}f(a) + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + \frac{1}{2}f(b) \right]$
 Error of the trapezoid rule: $|\epsilon| \leq h^2 \frac{b-a}{12} \max_{a \leq x \leq b} |f''(x)|$.
- Simpson rule: $\int_a^b f(x) dx \approx \frac{h}{3} [f_0 + 4f_1 + 2f_2 + 4f_3 + \dots + 2f_{n-2} + 4f_{n-1} + f_n]$
 with $f_i = f(x_i)$.
 Error of the Simpson rule: $|\epsilon| \leq h^4 \frac{b-a}{180} \max_{a \leq x \leq b} |f^{(4)}(x)|$.
- Gauss-Seidel iteration: $\mathbf{x}^{(k+1)} = \mathbf{b} - \mathbf{L}\mathbf{x}^{(k+1)} - \mathbf{U}\mathbf{x}^{(k)}$ with $\mathbf{A} = \mathbf{I} + \mathbf{L} + \mathbf{U}$.
- Jacobi iteration: $\mathbf{x}^{(k+1)} = \mathbf{b} + (\mathbf{I} - \mathbf{A})\mathbf{x}^{(k)}$
- Euler method: $\mathbf{y}_{n+1} = \mathbf{y}_n + h\mathbf{f}(x_n, \mathbf{y}_n)$
- Improved Euler method: $\mathbf{k}_1 = h\mathbf{f}(x_n, \mathbf{y}_n)$, $\mathbf{k}_2 = h\mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{k}_1)$,
 $\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{2}\mathbf{k}_1 + \frac{1}{2}\mathbf{k}_2$.
- Classical Runge-Kutte method:
 $\mathbf{k}_1 = h\mathbf{f}(x_n, \mathbf{y}_n)$, $\mathbf{k}_2 = h\mathbf{f}(x_n + h/2, \mathbf{y}_n + \mathbf{k}_1/2)$,
 $\mathbf{k}_3 = h\mathbf{f}(x_n + h/2, \mathbf{y}_n + \mathbf{k}_2/2)$, $\mathbf{k}_4 = h\mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{k}_3)$,
 $\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{6}\mathbf{k}_1 + \frac{1}{3}\mathbf{k}_2 + \frac{1}{3}\mathbf{k}_3 + \frac{1}{6}\mathbf{k}_4$.
- Backward Euler method: $\mathbf{y}_{n+1} = \mathbf{y}_n + h\mathbf{f}(x_{n+1}, \mathbf{y}_{n+1})$
- Finite differences: $\frac{\partial u}{\partial x}(x, y) \approx \frac{u(x+h, y) - u(x-h, y)}{2h}$
 $\frac{\partial^2 u}{\partial x^2}(x, y) \approx \frac{u(x+h, y) - 2u(x, y) + u(x-h, y)}{h^2}$
 $\frac{\partial u}{\partial y}(x, y) \approx \frac{u(x, y+h) - u(x, y-h)}{2h}$
 $\frac{\partial^2 u}{\partial y^2}(x, y) \approx \frac{u(x, y+h) - 2u(x, y) + u(x, y-h)}{h^2}$