

LAPLACE TRANSFORMS : GENERAL FORMULAS

Formula	Name, Comments
$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st}f(t) dt$ $f(t) = \mathcal{L}^{-1}\{F(s)\}$	<p>Definition of Transform</p> <p>Inverse Transform</p>
$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$	Linearity
$\mathcal{L}\{e^{at}f(t)\} = F(s - a)$ $\mathcal{L}^{-1}\{F(s - a)\} = e^{at}f(t)$	<p><i>s</i>-Shifting (First Shifting Theorem)</p>
$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$ $\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$ $\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{(n-1)}f(0) - \dots$ $\dots - f^{(n-1)}(0)$ $\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}\mathcal{L}(f)$	<p>Differentiation of Function</p> <p>Integration of Function</p>
$(f * g)(t) = \int_0^t f(\tau)g(t - \tau) d\tau$ $= \int_0^t f(t - \tau)g(\tau) d\tau$ $\mathcal{L}(f * g) = \mathcal{L}(f)\mathcal{L}(g)$	Convolution
$\mathcal{L}\{f(t - a)u(t - a)\} = e^{-as}F(s)$ $\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t - a)u(t - a)$	<p><i>t</i>-Shifting (Second Shifting Theorem)</p>
$\mathcal{L}\{tf(t)\} = -F'(s)$ $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(\tilde{s})d\tilde{s}$	<p>Differentiation of Transform</p> <p>Integration of Transform</p>
$\mathcal{L}(f) = \frac{1}{1 - e^{-ps}} \int_0^p e^{-st}f(t) dt$	<i>f</i> Periodic with Period <i>p</i>