

Exercise 1. The following functions are assumed to be zero outside the given interval. Represent them using unit step functions and find their Laplace transforms.

a. $t \quad (0 < t < 2)$

b. $\exp\left(-\frac{\pi}{2}t\right)$

Exercise 2. (Time shifting) Find $f(t)$ if

$$\mathcal{L}(f) = \frac{4(1 - e^{-\pi s})}{s^2 + 4}$$

Exercise 3. (IVP, unit step functions) Using the Laplace transform, solve

$$y'' + y = t \quad \text{if } 0 < t < 1, \quad \text{and } 0 \quad \text{if } t > 1, \\ y(0) = 0, \quad y'(0) = 0$$

Exercise 4. (IVP, delta function) Using the Laplace transform, solve

$$y'' + 4y = \delta(t - \pi) \\ y(0) = 8, \quad y'(0) = 0$$

Exercise 5. Convolution.

a. Find $e^{-t} \star e^t$

b. Using the convolution theorem, find $f(t)$ if

$$\mathcal{L}(f) = \frac{2\pi s}{(s^2 + \pi^2)^2}$$