

Exercise 9. Determine and sketch the or graph the sets in the complex plane given by:

a. $|z + 1 - 2i| \leq 3$

b. $\operatorname{Re}\left(\frac{1}{z}\right) < 1$

Exercise 10. Find $\operatorname{Re}f$ and $\operatorname{Im}f$ of $f(z) = (z - 1)/(z + 1)$, and their values at $2i$.

Exercise 11. Let

$$f(z) = \begin{cases} 0 & \text{for } z = 0, \\ (\operatorname{Im}z^2)/|z^2| & \text{for } z \neq 0. \end{cases}$$

Find out whether $f(z)$ is continuous at $z = 0$.

Exercise 12. Let

$$f(z) = \begin{cases} 0 & \text{for } z = 0, \\ \operatorname{Re}z/(1 - |z|) & \text{for } z \neq 0. \end{cases}$$

Find out whether $f(z)$ is continuous at $z = 0$.

Exercise 13. Find the value of the derivative of $(z - 2i)^3$ at $5 + 2i$.

Exercise 14. Are the following functions analytic? (Use Cauchy-Riemann equations).

a. $f(z) = e^{-x}\cos(y) - ie^{-x}\sin(y)$

b. $f(z) = \operatorname{Re}(z^2) - i\operatorname{Im}(z^2)$

c. $f(z) = -i/z^4$.

Exercise 15. Are the following functions harmonic? If yes, find a corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

a. $u = -2xy$

b. $v = e^{-x} \sin(2y)$

Exercise 16. Determine a so that the function

$u = e^{-\pi x} \cos(\alpha y)$ is harmonic and find a harmonic conjugate.

Exercise 17. Find e^z in the form $u + iv$ and $|e^z|$ if z equals

a. $3 + 4i$

b. $1 - 3\pi i$

Exercise 18. Find the real and the imaginary part of $\exp(-z^2)$

Exercise 19. Find all the solutions of $e^z = 1$.

Exercise 20. Find $\sin\left(\frac{\pi}{4}i\right)$ and $\cos\left(\frac{\pi}{2} - \frac{\pi}{4}i\right)$ in the form $u + iv$.

Exercise 21. Find the value of $\cos(5 - 2i)$.

Exercise 22. Find $\text{Ln}z$ when z equals

a. -7

b. $0.6 - 0.8i$

Exercise 23. Find all the values of $\ln(e^i)$.

Exercise 24. Solve $\ln z = 0.4 + 0.2i$

Exercise 25. Find the principal the principal value of $(1 + i)^{1-i}$