

Exercise 1. Let $z_1 = -2 + 5i$, $z_2 = 3 - i$. Find, in the form $x + iy$:

- a. $z_1 z_2, \overline{(z_1 z_2)}$
- b. $\operatorname{Re}(z_1^2), (\operatorname{Re} z_1)^2$
- c. $\operatorname{Im}(1/z_2^2)$
- d. $z_1/z_2, \overline{z_2}/\overline{z_1}$
- e. $4(z_1 + z_2)/(z_1 - z_2)$

Exercise 2. Let $z = x + iy$. Find, in terms of x and y :

- a. $\operatorname{Im} \frac{1}{z}, \operatorname{Im} \left(\frac{1}{z^2} \right)$
- b. $\operatorname{Re}(z/\bar{z}), \operatorname{Im}(z/\bar{z})$

Exercise 3. Represent in polar form and graph in the complex plane:

- a. $2, 3i, 1 + i, -2 + 5i$
- b. $\frac{\sqrt{2} + i/3}{-\sqrt{8} - 2i/3}$
- c. $1 + \frac{1}{2}\pi i$

Exercise 4. Determine the principal value of the argument:

- a. $1 - i$
- b. $\sqrt{3} + i$
- c. $(1 - i)^{20}$

Exercise 5. Graph in the complex plane and represent in the form $x + iy$:

- a. $4\left(\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}\right)$
- b. $\sqrt{8}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

Exercise 6. Find and graph all the roots in the complex plane:

a. $\sqrt[3]{1 - i}$

b. $\sqrt[3]{343}$

c. $\sqrt[4]{-4}$

d. $\sqrt[4]{i}$

e. $\sqrt[5]{-1}$

Exercise 7 (optional). Solve $z^2 - (6 - 2i)z + 17 - 6i = 0$.

Exercise 8 (optional). Parallelogram inequality:

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2).$$

Prove and explain the name.