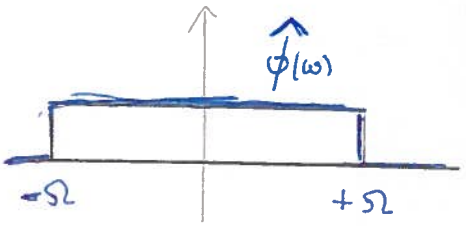
INPUT: $f(t)$

OUTPUT: $(f * \phi)(t)$

Ex. Low Pass Filter

$$\phi(t) = \frac{1}{\pi}\,\frac{\sin(\Omega t)}{t}$$

$$\hat{\phi}(\omega) = \begin{cases} \frac{1}{\sqrt{2\pi}}, & \text{if } |\omega| < \Omega \\ 0, & \text{if } |\omega| > \Omega \end{cases}$$



(For example, $\Omega = 20.000$ Hz)

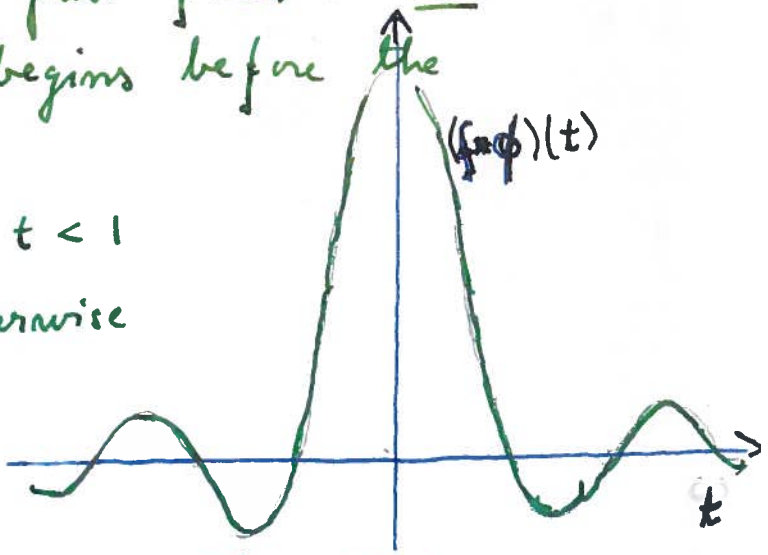
$$\widehat{(f * \phi)}(\omega) = \sqrt{2\pi}\,\hat{f}(\omega)\,\hat{\phi}(\omega)$$

$$= \begin{cases} \hat{f}(\omega), & \text{if } |\omega| \leq \Omega \\ 0, & \text{if } |\omega| > \Omega \end{cases}$$

All frequencies $|\omega| > \Omega$ are filtered away from the output signal!

Unfortunately, the low pass filter is not causal: the output begins before the signal has arrived!

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$



The output signal $(f * \phi)(t)$

Butterworth filter (CAUSAL)

$$\phi(t) = \begin{cases} Ae^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$