Oppgaver til øving 11

October 29, 2020

15.3

15.3:4

Show that

$$\frac{1}{(1-z)^2} = \sum_{n=0}^{\infty} (n+1) z^n$$

(a) by using the Cauchy product, (b) by differentiating a suitable series.

15.3:8

Find the radius of convergence of

$$\sum_{n=1}^{\infty} \frac{5^n}{n(n+1)} z^n$$

in two ways: (a) directly by the Cauchy–Hadamard formula in Sec. 15.2, and (b) from a series of simpler terms by using Theorem 3 or Theorem 4.

15.3:17

Show that if $f(z) = \sum_{n=0}^{+\infty} a_n z^n$ and f is an odd function $((f(-z) = -f(z))$, then $a_n = 0$ for n even.

15.4

15.4:5

Find the Maclaurin series and its radius of convergence for

$$\frac{1}{2+z^4}.$$

15.4:8

Find the Maclaurin series and its radius of convergence for

$$\sin^2 z.$$

15.4:24

Find the Taylor series centered at $z_0 = 1$ of

$$e^{z(z-2)}$$

and its radius of convergence.

15.5

15.5:10

Prove that the series

$$\sum_{n=0}^{+\infty} \frac{z^{2n}}{2n!}$$

converges uniformly in $|z| \leq 10^{20}$.

Review chapter 15

Exercise 26

Find the Taylor series centered at i of

$$z^4$$

and its radius of convergence.