TWILIGHT ZONE.

$$\mathcal{F}\{\delta(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta(x) e^{-i\omega x} \, dx = \frac{1}{\sqrt{2\pi}} e^{-i\omega \cdot 0}$$

DIRAC'S δ

$$= \frac{1}{\sqrt{2\pi}}$$

!

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega x} \, d\omega$$

(INVERSE)

#

PLANCHEREL

- $$\boxed{\int_{-\infty}^{\infty} |f(x)|^2\,dx = \int_{-\infty}^{\infty} |\hat{f}(\omega)|^2\,d\omega}$$

Counterpart to Parseval's formula.

Verification: $|\hat{f}(\omega)|^2 = \hat{f}(\omega)\,\overline{\hat{f}(\omega)}$

$$\int |\hat{f}(\omega)|^2\,d\omega = \int \hat{f}(\omega)\,\overline{\hat{f}(\omega)}\,d\omega$$

conjugate

$$= \int_{-\infty}^{\infty} \hat{f}(\omega)\,\overline{\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\,e^{-i\omega x}\,dx\right)}\,d\omega$$

$$= \int_{-\infty}^{\infty} \hat{f}(\omega)\,\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \overline{f(x)}\,e^{+i\omega x}\,dx\,d\omega$$

$\overline{a+b} = \bar{a}+\bar{b}$

$\overline{ab} = \bar{a}\cdot\bar{b}$

$$= \int_{-\infty}^{\infty} \overline{f(x)}\,\underbrace{\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(\omega)\,e^{+i\omega x}\,d\omega\right)}_{\text{INV. TRANSF.} = f(x)}\,dx = \int_{-\infty}^{+\infty} |f(x)|^2\,dx$$

PARTIAL DIFFEREN-TIAL EQS

- $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

  LAPLACE 1787

- $$\frac{\partial^2 v}{\partial t^2} = c^2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

  WAVE EQN 1746
  d'ALEMBERT

- $$\frac{\partial v}{\partial t} = k\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

  HEAT EQN
  DIFFUSION EQN 1822

  FOURIER

In <u>one</u> space variable x

- $u_{xx} = 0 \quad , \quad u(x) = Ax + B$ LAPLACE EQN
- $\dfrac{\partial^2 v}{\partial t^2} = c^2 \dfrac{\partial^2 v}{\partial x^2} \quad , \quad v = v(x, t)$ WAVE EQN
- $\dfrac{\partial v}{\partial t} = k \dfrac{\partial^2 v}{\partial x^2} \quad , \quad v = v(x, t)$ HEAT EQN

$\underline{k > 0}$

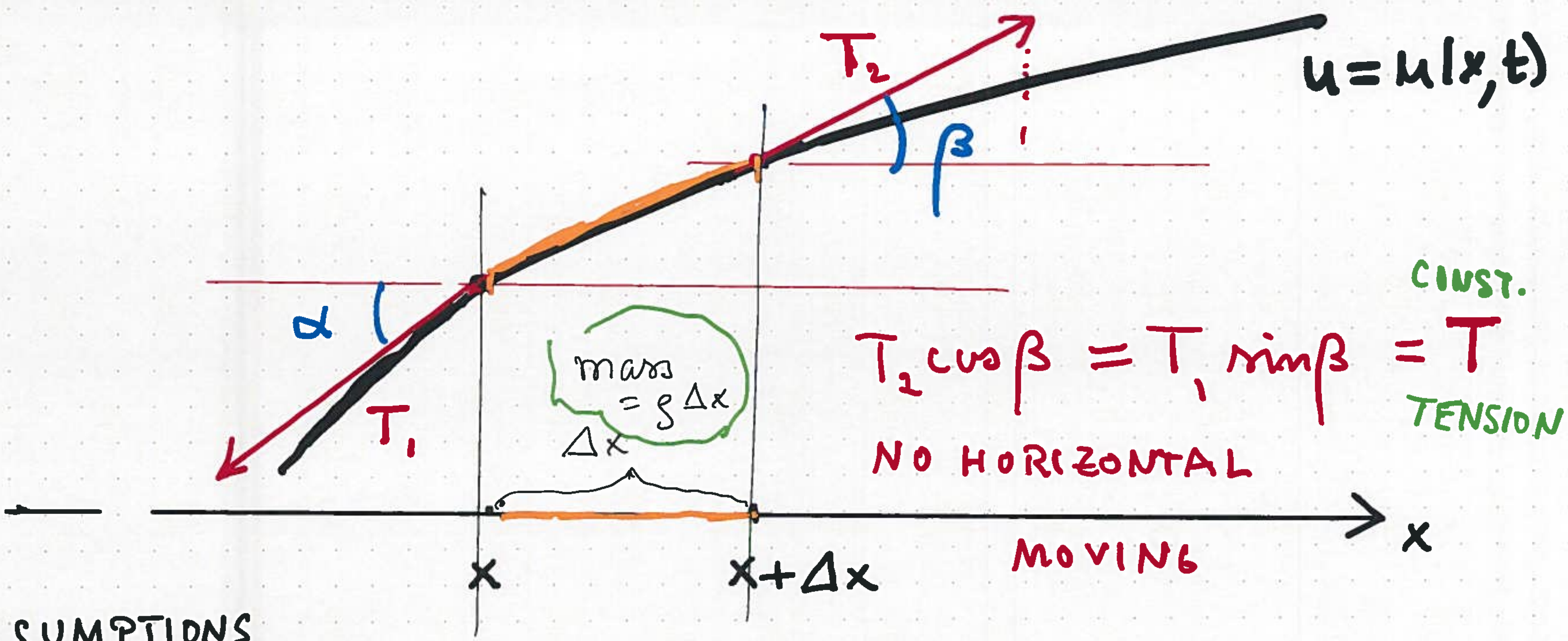
<u>Ex:</u> $u_{xy} = x + y^2 \quad , \quad u = u(x, y)$

$$u_{xy} = \frac{\partial^2 u}{\partial x \partial y}$$

$$u_x = xy + \frac{y^3}{3} + C(x)$$

$$u = \frac{x^2 y}{2} + \frac{y^3 x}{3} + \underbrace{\int C(x)\,dx}_{c(x)} + a(y)$$

VIBRATING STRING



ASSUMPTIONS

(1) Perfectly elastic, and
ρ = constant [g/cm]

(2) No external forces (like gravitation)

(3) Vertical motion in a plane; no

horizontal motion.

$$\begin{cases} T_1 \cos\alpha = T_2 \cos\beta = T \quad (\text{const.}) \\ T_2 \sin\beta - T_1 \sin\alpha = \underbrace{\rho\,\Delta x}_{\Delta m} \cdot \dfrac{\partial^2 u}{\partial t^2} \end{cases}$$

acceleration

$\bar{F} = m\bar{a}$

Newton II

Divide the last eqn by T to get

$$\frac{\cancel{T_2} \sin\beta}{\underbrace{T}_{\cancel{T_2} \cos\beta}} - \frac{\cancel{T_1} \sin\alpha}{\underbrace{T}_{\cancel{T_1} \cos\alpha}} = \rho\,\Delta x\, u_{tt} \cdot \frac{1}{T}$$

$$\tan\beta - \tan\alpha = \rho\,\Delta x\, u_{tt} \cdot \frac{1}{T}$$

$$\frac{u_x(x+\Delta x, t) - u_x(x, t)}{\Delta x} = \frac{\rho}{T} u_{tt}$$

Let $\Delta x \to 0$.

$$u_{xx} = \frac{\rho}{T} u_{tt}$$

ONE DIMENSIONAL WAVE EQN.

$u = u(x,t)$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = \frac{T}{\rho} \frac{\partial^2 u}{\partial x^2} \; ; \; c^2 = \frac{T}{\rho}}$$

<u>Comment:</u> As we shall see (d'Alembert's formula), c is the <u>speed</u> of propagation.

- Strong tension $T \Rightarrow$ Fast speed c
- Heavy string (large density ρ) $\Rightarrow$ Slow speed c.