PROBLEM SET 11.4

1. **(Calculus review)** Review complex numbers.
2. **(Even and odd functions)** Show that the complex Fourier coefficients of an even function are real and those of an odd function are pure imaginary.
3. **(Fourier coefficients)** Show that
$a_0 = c_0,\ a_n = c_n + c_{-n},\ b_n = i(c_n - c_{-n})$.
4. Verify the calculations in Example 1.
5. Find further terms in (9) and graph partial sums with your CAS.
6. Obtain the real series in Example 1 directly from the Euler formulas in Sec. 11.

7–13 COMPLEX FOURIER SERIES

Find the complex Fourier series of the following functions. (Show the details of your work.)

7. $f(x) = -1$ if $-\pi < x < 0$, $f(x) = 1$ if $0 < x < \pi$
8. Convert the series in Prob. 7 to real form.
9. $f(x) = x \quad (-\pi < x < \pi)$
10. Convert the series in Prob. 9 to real form.
11. $f(x) = x^2 \quad (-\pi < x < \pi)$
12. Convert the series in Prob. 11 to real form.
13. $f(x) = x \quad (0 < x < 2\pi)$

14. **PROJECT. Complex Fourier Coefficients.** It is very interesting that the c_n in (6) can be derived directly by a method similar to that for a_n and b_n in Sec. 11.1. For this, multiply the series in (6) by e^{-imx} with fixed integer m, and integrate termwise from $-\pi$ to π on both sides (allowed, for instance, in the case of uniform convergence) to get

$$\int_{-\pi}^{\pi} f(x)e^{-imx}\,dx = \sum_{n=-\infty}^{\infty} c_n \int_{-\pi}^{\pi} e^{i(n-m)x}\,dx.$$

Show that the integral on the right equals 2π when $n = m$ and 0 when $n \neq m$ [use (3b)], so that you get the coefficient formula in (6).