Problem 5 Determine the radius of convergence of the Taylor series of the function

$$h(z) = \frac{2}{1 + \cosh(z)}$$

centered at the origin. Explain your answer.

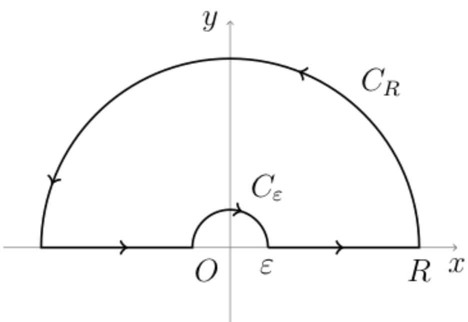
Problem 6 Consider the function

$$f(z) = \frac{\ln(z)}{z^2 + a^2}, \quad a > 0,$$

in the upper half plane. For $z = re^{i\theta}, 0 \leq \theta \leq \pi$ we define

$$\ln(z) = \ln(r) + i\theta.$$

Let C be the following contour:



a) Determine $\oint_C f(z)\, dz$ by residue calculations.

b) Show that the integrals over the half circles C_R and C_ε approach zero as $R \to \infty$ and $\varepsilon \to 0$.

c) Compute

$$\int_0^\infty \frac{\ln(x)\, dx}{x^2 + a^2}.$$