Unique Analytic Continuation (example)
Assume that $f(z)$ and $g(z)$ are analytic functions in the disc $|z| < 10$ which are equal on the real interval given by $\{x + iy | y = 0,\ x \in (0, 0.001)\}$.
In other words, $f(x + 0i) = g(x + 0i)$ when $0 < x < 0.001$.

Prove that $f(z) = g(z)$ for all z in the disc.

[Hint: $f(z) - g(z)$ is an analytic function with many zeros.]