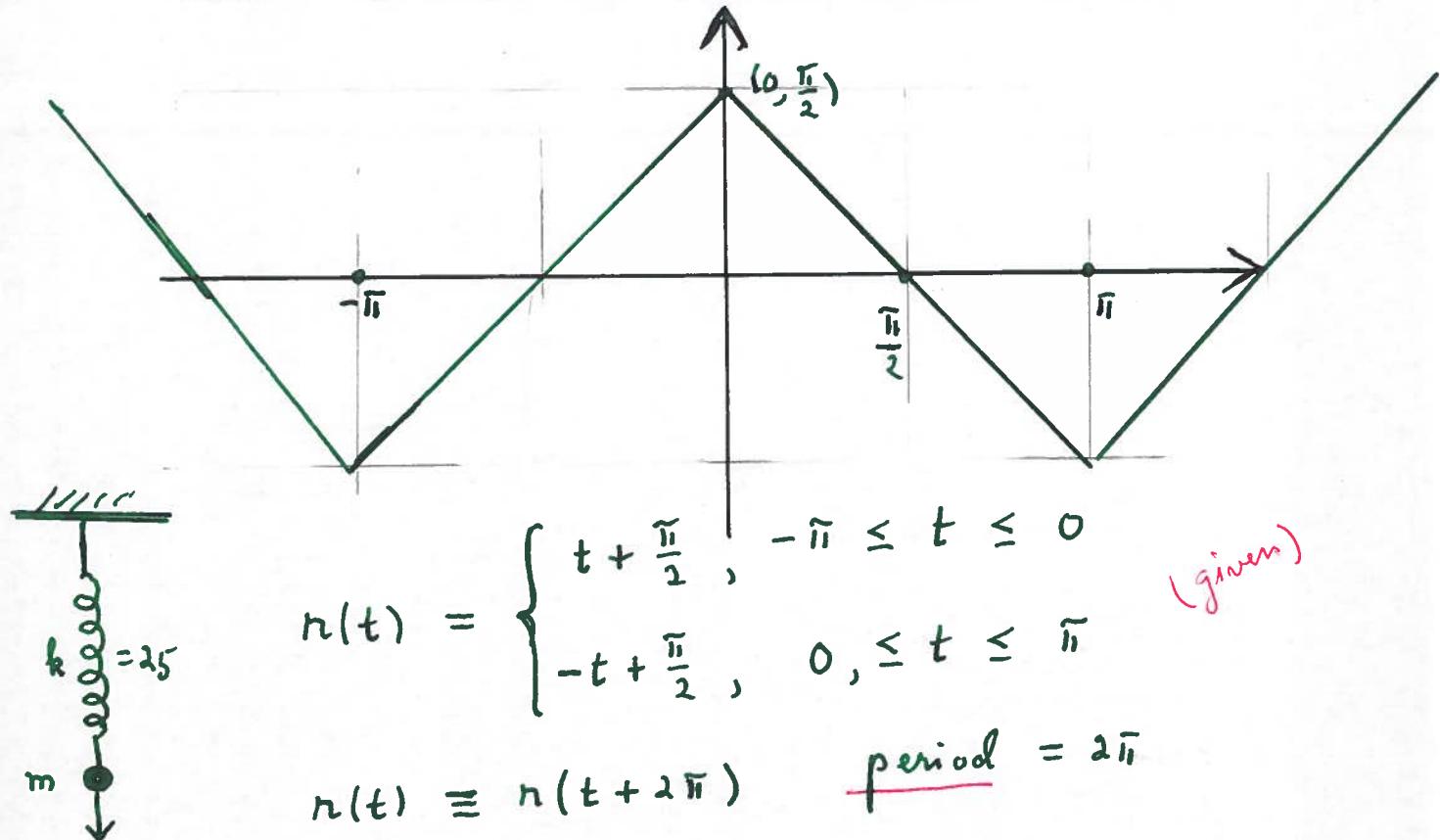


EXAMPLE (WITH INTERFERENCE)

$$\boxed{\frac{d^2y}{dt^2} + 0,02 \frac{dy}{dt} + 25y = n(t)}$$

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Notice that the free mass-spring system, i.e.
 $n(t) \equiv 0$, has the general solution

$$y(t) = e^{-\frac{t}{100}} \underbrace{(A \cos(4,99999t) + B \sin(4,99999t))}_{= C \cos(4,99999t - \delta)},$$

OBS! $\approx 5t$ EXACT

which is easy to find. As we shall see, the dominating term in the general solution (with $n(t)$ as above) is

$$0,5092958... \sin(5t).$$

① First, we expand the forcing term as a Fourier series:

$$n(t) = \frac{4}{\pi} \left(\cos(t) + \frac{1}{3^2} \cos(3t) + \frac{1}{5^2} \cos(5t) + \dots \right)$$

② Then we solve (only one term from $n(t)$!)

$$y'' + 0,02y' + 25y = \frac{4}{\pi n^2} \cos(nt), \quad n=odd$$

using the ANSATZ:

$$\begin{cases} y = A_n \cos(nt) + B_n \sin(nt) \\ y' = -nA_n \sin(nt) + nB_n \cos(nt) \\ y'' = -n^2 y \end{cases}$$

Inserting, we have

$$[25 - n^2] [A_n \cos(nt) + B_n \sin(nt)] + \\ + 0,02n [-A_n \sin(nt) + B_n \cos(nt)] = \frac{4}{\pi n^2} \cos(nt).$$

The terms cancel if

$$\begin{cases} (25 - n^2)A_n + 0,02nB_n = \frac{4}{\pi n^2} \\ -0,02nA_n + (25 - n^2)B_n = 0 \end{cases}$$

linear system: A_n, B_n
unknowns: A_n, B_n

Hence, writing $D_n = (25 - n^2)^2 + (0,02n)^2$,

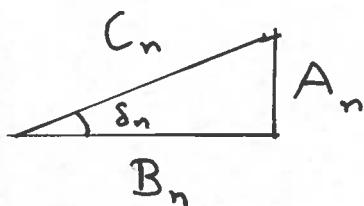
$$A_n = \frac{4(25 - n^2)}{n^2 \pi D_n}, \quad B_n = \frac{0,08}{n \pi D_n}$$

$$n = 1, 3, 5, 7, 9, \dots$$

③ Superposition yields the final solution

$$y(t) = e^{-t/100} [A \cos(4,99999t) + B \sin(4,99999t)]$$

$$+ \sum_{n=1}^{\infty} \underbrace{A_n \cos(nt) + B_n \sin(nt)}_{C_n \sin(nt - \delta_n)}$$



$$C_n = \sqrt{A_n^2 + B_n^2} \quad (\text{= amplitude})$$

$$C_1 \approx 0,0530 \quad (C_{2n} = 0)$$

$$C_3 \approx 0,0088$$

$$\underline{C_5 \approx 0,5093!} \quad \text{This dominates.}$$

$$C_7 \approx 0,0011$$

The interference comes from

$$C_9 \approx 0,0003$$

$$5t \approx 4,99999t$$

$$C_{11} \approx 0,0001 \quad (\text{The term } n=5 \text{ is}$$

$$\dots 0,5092558\dots \sin(5t).)$$

$$A_5 = 0, B_5 = \frac{8}{5\pi}$$