We say that signal $\hat{f}(t)$ is band-limited to $(-L, L)$ if it has representation

$$\hat{f}(t) = \sum_{n=-\infty}^{\infty} \hat{f}(n) e^{i\omega t} dt$$

(1)

Shannon theorem gives precise reconstruction of such signal from its samples at the points $\{\frac{n\omega}{L} \}_{n=-\infty}^{\infty}$.

Here how this works:

1) Write the Fourier series for the function $\hat{f}(\omega)$ on $[-L, L]$

$$\hat{f}(\omega) = \sum_{n=-\infty}^{\infty} c_n e^{i \frac{n\omega}{L}}$$

2) Substitute this series in (1):

$$\hat{f}(t) = \sum_{n=-\infty}^{\infty} c_n \int_{-L}^{L} e^{i\omega t + \frac{i n\omega}{L}} dt$$

and evaluate the integrals in the right-hand side:

$$\int_{-L}^{L} e^{i\omega t + \frac{i n\omega}{L}} dt = 2 \cdot \frac{\sin(\frac{Lt + n\pi}{L})}{t + \frac{n\pi}{L}}$$

So

$$\hat{f}(t) = 2 \sum_{n=-\infty}^{\infty} c_n \frac{\sin(\frac{Lt + n\pi}{L})}{t + \frac{n\pi}{L}}$$

(2)
3) Expression for the coefficients of the Fourier series:

\[ c_n = \frac{1}{2L} \int_{-L}^{L} f(t) e^{-i \frac{n \pi t}{L}} dt \]

Compare this with (1) gives \( c_n = \frac{1}{2L} \hat{f} \left( \frac{n \pi}{L} \right) \)

We substitute this in (2)

\[ \hat{f} (t) = \sum_{n} \hat{f} \left( \frac{n \pi}{L} \right) \frac{\sin \left( \frac{L t + n \pi}{L} \right)}{L t + n \pi} \]

This is the famous Shannon formula!

4) Remarks:
1. Distance between sampling points is \( \frac{\pi}{L} \), it is called Nyquist rate, bigger \( L \) more often should we sample.

2. In real applications people use oversampling i.e. sample more often in order to provide better stability.