EXAMPLES 2018

Peter Lindqvist

(1) We know that the solutions of the problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = k \dfrac{\partial^2 u}{\partial t^2} & (0 < t < \pi, \ t > 0) \\ u(0,t) = 0 = u(\pi, t) & (t > 0) \end{cases}$$

are all given by the expression

$$\sum_{n=1}^{\infty} c_n \sin(nx)\, e^{-n^2 k t} .$$

Solve the Heat Equation $\dfrac{\partial v}{\partial t} = k \dfrac{\partial^2 v}{\partial t^2}$ with

$$\begin{cases} \text{initial temperature } v(x,0) = \dfrac{3}{2} \quad (0 < x < \pi), \\ \text{end points kept at} \\ \qquad u(0,t) = 1, \ u(\pi, t) = 2 \quad (t > 0) \end{cases}$$

(2)

$$\begin{cases} y''(t) + 2y'(t) + 2y(t) = \delta(t-3), \\ y(0) = 0, \ y'(0) = 0. \end{cases}$$

↑ DIRAC's DELTA

(3) The Wave Eqn has the solution

$$v(x,t) = x^2 - c^2t^2 + \frac{1}{2c} \int_{x-ct}^{x+ct} e^{-\xi^2} d\xi .$$

First, split $x^2 - c^2t^2$ as

$$x^2 - c^2t^2 = \frac{f(x-ct) + f(x+ct)}{2} .$$

Find $\left. \dfrac{\partial v(x,t)}{\partial t} \right|_{t=0} = \, ?$

④ Draw the curve

$$|z-1||z+1| = 1 \qquad \text{(Bernoulli's lemniscate)}$$

in the complex plane.

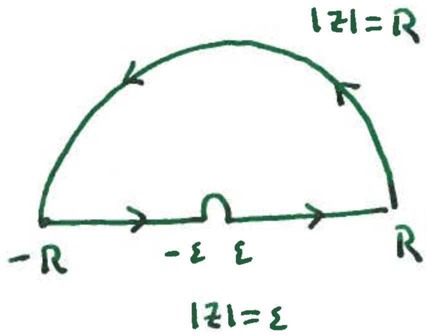
⑤ Find some number $R > 0$ such that

$$\frac{11}{10+e^x} = 1 + \sum_{n=1}^{\infty} a_n x^n$$

converges at least for $-R < x < R$.

⑥ Evaluate the integral

$$\int_0^\infty \frac{\log(x)}{x^2+3^2}\,dx$$



<u>Hint:</u> $\log(z) = |z| + i\theta$,

when $0 \le \theta \le \pi$, $z = re^{i\theta}$.

⑦ Determine the real constants a, b and the function $v(x,y)$ so that

$$f(z) = ax^2 + by^2 + i\,v(x,y), \qquad i^2 = -1$$

is analytic!

⑧ $i^{-i} = ?$ ⑨ $\displaystyle\oint_{|z|=2018} z^2 e^{\frac{1}{z}}\,dz = ?$