a) Calculate Laplace’s integrals
\[ \int_{-\infty}^{\infty} \frac{e^{ix}}{x \pm ia} \, dx, \quad a > 0 \]
by using the contour consisting of the half-circle \( |z| = R, y > 0 \) and the diameter \(-R \leq x \leq R, y = 0\). (Let \( R \to \infty \) and show that the integrals on the half-circle approach zero.) Then evaluate the integral
\[ \int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + a^2} \, dx \]
by adding the two integrals.

b) Find a conformal mapping \( w = f(z) \) that maps the right half-plane \( y > 0 \) onto the unit disc \(|w| < 1\). We require that
\[ i = f(i), \quad -1 = f(0), \quad -i = f(-i). \]

Hint: The mapping must be of the form
\[ w = \frac{az + b}{cz + d}. \]