a) Calculate Laplace's integrals

$$\int_{-\infty}^{\infty} \frac{e^{ix}}{x \pm ia}\, dx, \quad a > 0$$

by using the contour consisting of the half-circle $|z| = R$, $y > 0$ and the diameter $-R \leq x \leq R$, $y = 0$. (Let $R \to \infty$ and show that the integrals on the half-circle approach zero.) Then evaluate the integral

$$\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + a^2}\, dx$$

by adding the two integrals.

b) Find a conformal mapping $w = f(z)$ that maps the right half-plane $y > 0$ onto the unit disc $|w| < 1$. We require that

$$i = f(i), \quad -1 = f(0), \quad -i = f(-i).$$

Hint: The mapping must be of the form

$$w = \frac{az + b}{cz + d}.$$