NTNU

Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4120 Matematikk 4K**

Academic contact during examination: Eduardo Ortega

Phone: 73591799

Examination date: 13 August 2019

Examination time (from–to): 09:00–13:00

Permitted examination support material: C: Bestemt, enkel kalkulator tillatt.

Other information:
The seven problems 1, 2, 3, 4, 5, 6a, 6b have equal weight

Language: English

Number of pages: 2

Number of pages enclosed: 3

Checked by:

Informasjon om trykking av eksamensoppgave	
Originalen er:	
1-sidig ☒	**2-sidig** ☐
sort/hvit ☒	**farger** ☐
skal ha flervalgskjema ☐	

Date Signature

Problem 1 Solve the initial value problem

$$\begin{cases} \frac{d^2y(t)}{dt^2} - 4\frac{dy(t)}{dt} + 13y(t) = 4\delta(t-3) \\ y(0) = 0 \; y'(0) = 0 \end{cases}$$

where $\delta(t-3)$ is Dirac's delta centered at $t = 3$.

Problem 2 Determine the radius of convergence for the power series

$$\sum_{n=1}^{\infty} n^2 3^n z^n$$

.

Problem 3 Evaluate Cauchy's Principal Value

$$\mathrm{PV} \int_{-\infty}^{+\infty} \frac{dx}{(x^2-1)(x^2+1)}.$$

Problem 4 Let $u = u(x,t)$ represent the temperature in a rod of length π. The endpoints are kept at temperature zero. The rod is not totally insulated, and the temperature satisfies the following equation:

$$\begin{cases} u_t(x,t) + u(x,t) = u_{xx}(x,t) & (0 < x < \pi, \; t > 0) \\ u(0,t) = 0 = u(\pi,t). \end{cases}$$

Given the initial temperature

$$u(x,0) = \sum_{n=1}^{\infty} (-1)^n \frac{\sin(2nx)}{n^2},$$

find the temperature $u(x,t)$ by separating the variables.

Problem 5 Evaluate Fresnel's integrals

$$\int_0^{\infty} \cos(x^2)\, dx \quad \int_0^{\infty} \sin(x^2)\, dx?$$

by integrating the function

$$f(z) = e^{iz^2}$$

along the boundary of the sector

$$z = re^{i\theta},\ 0 < \theta < \frac{\pi}{4},\ 0 < r < R.$$

Problem 6

a) Find the poles of the function

$$f(z) = \frac{1 + e^{i\pi z}}{(z-1)^2(z+1)^2}$$

and determine their orders.

b) Calculate the residues of $f(z)$ at the poles.

x