SINE/COSINE TRANSFORMS

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\underbrace{\left(\cos(\omega x) - i\sin(\omega x)\right)}_{= e^{-i\omega x}}\,dx$$

$$= \underbrace{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cos(\omega x)\,dx}_{a(\omega)} - i\,\underbrace{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\sin(\omega x)\,dx}_{b(\omega)}$$

EVEN: $a(-\omega) = a(\omega)$ ODD: $b(-\omega) = -b(\omega)$

INVERSION:

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \underbrace{\left(a(\omega) - i\,b(\omega)\right)}_{\hat{f}(\omega)}\underbrace{\left(\cos(\omega x) + i\sin(\omega x)\right)}_{e^{+i\omega x}}\,d\omega$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \underbrace{a(\omega)\cos(\omega x)}_{\text{EVEN}}\,d\omega + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \underbrace{b(\omega)\sin(\omega x)}_{\text{EVEN}}\,d\omega + i\,\overbrace{(0+0)}^{\text{odd integrands}}$$

$$= \frac{2}{\sqrt{2\pi}}\int_{0}^{\infty} a(\omega)\cos(\omega x)\,d\omega + \frac{2}{\sqrt{2\pi}}\int_{0}^{\infty} b(\omega)\sin(\omega x)\,d\omega$$

$(= f(x))$ If we know $a(\omega)$, $b(\omega)$ for $\omega \geq 0$, we obtain $f(x)$

We renormalize $a(\omega)$ and $b(\omega)$ by a constant.

$$\begin{cases} A(\omega) = \dfrac{1}{\pi}\displaystyle\int_{-\infty}^{\infty} f(x)\cos(\omega x)\,dx \\[2ex] B(\omega) = \dfrac{1}{\pi}\displaystyle\int_{-\infty}^{\infty} f(x)\sin(\omega x)\,dx \end{cases}$$

THE COSINE TRANSFORM

THE SINE TRANFORM

$$f(x) = \int_0^{\infty} \big(A(\omega)\cos(\omega x) + B(\omega)\sin(\omega x)\big)\,d\cancel{x}\omega$$

INVERSE