Problems on conformal mappings. (make pictures!).
(some of them may be complicated ☺)

1. a. Find the image of the circle $\{z : |z - \frac{1}{2}| = \frac{1}{2}\}$ under the mapping

$$z \mapsto w(z) = \frac{1}{z-1} \qquad (*)$$

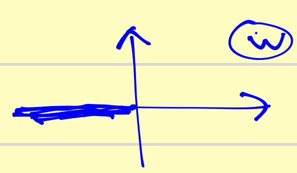
b. Find the image of the disk $\{z : |z - \frac{1}{2}| < \frac{1}{2}\}$ under the maping $(*)$

c. Find the image of the exterior of the disk: $\{z : |z - \frac{1}{2}| > \frac{1}{2}\}$ under the mapping $(*)$

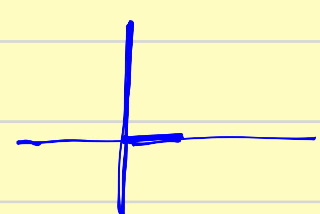
d. Find the image of the domain between two circles $\{z : \frac{1}{2} < |z - \frac{1}{2}|$ and $1 > |z-1|$ under the mapping $(*)$.

2. Find a conformal mapping of the $\mathbb{C}_+$ (upper halfplane) onto

a) $\{w,\ w \in (-\infty, 0]\}$



b) $\{w,\ \mathrm{Re}\, w > 0;\ w \notin [0,1]\}$



3). Find a conformal mapping of the unit disk onto itself such that $f(0) = 1/2$.

4) a) Find the images of the following curves under the mapping $z \mapsto w = z^2$:

- $\{z : \arg z = \alpha\}$, for some fixed $\alpha \in (-\pi, \pi)$
- $\{z : \mathrm{Re}\, z = a\}$ for some fixed $a \in \mathbb{R}$

- $\{z : \operatorname{Im} z = a\}$ for some fixed $a \in \mathbb{R}$

4) Find the images under the mapping $z \mapsto w = z^2$ of the following domains:

- $\{z : \operatorname{Im} z > 0\}$
- $\{z : |z| < 2\}$
- $\{z ; \operatorname{Re} z > 1\}$
- $\{z ; |z| > 1/2, \operatorname{Re} z > 0\}$

5) Let the value of the function $w = z^{1/2}$ is chosen by the relation $z^{1/2}\big|_{z=-1} = -i$.

Find the images of the domains

- $\{z : z \notin [0, \infty]\}$
- $\{z : \operatorname{Im} z^2 > 2 \operatorname{Re} z + 1\}$

6. Find a conformal mapping of the upper half plane $\mathbb{C}_+$ onto the strip $\{z : |\operatorname{Re} z| < 1\}$

7). Find a conformal mapping of $\mathbb{C}_+$ onto the half strip $\{z : |\operatorname{Re} z| < 1, \operatorname{Im} z > 0\}$.

8) Find a conformal mapping of the disk with a cut: $\{z : |z| < 1, z \in [\frac{1}{2}, 1]\}$ onto the disk $\{w : |w| < 1\}$.

9) Find a mapping of the unit disk $\mathbb{D} = \{z : |z| < 1\}$ onto the plane with infinite cut $\{w, w \in [1, \infty)\}$.

10) Find a mapping of the unit disk onto the complex plane with finite cut $\overline{\mathbb{C}} \setminus [1,2] = \{ w \in \overline{\mathbb{C}},\ w \in [1,2] \}$

Comment: When we write $\overline{\mathbb{C}}$ we add infinity to the complex plane, so $\overline{\mathbb{C}} \setminus [1,2]$ is a domain without holes.