FIBONACCI (? 1175 – 1250)

1, 1, 2, 3, 5, 8, 13, 21, 34, ... $\qquad F_6 = 8$

GENERATING FUNCTION:

$$z + z^2 + 2z^3 + 3z^4 + 5z^5 + \cdots = \sum_{n=1}^{\infty} F_n z^n$$

$$= f(z) \qquad \boxed{F_n = F_{n-1} + F_{n-2}}$$

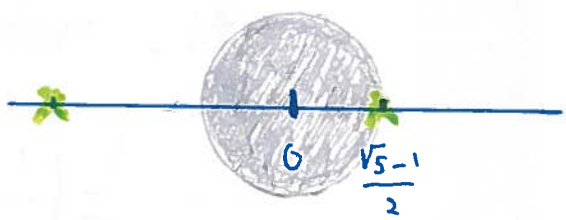
The defining rule.

$$z f(z) + z^2 f(z) = z^2 + 2z^3 + 3z^4 + \cdots$$

$$= f(z) - z$$

$$f(z) = \frac{z}{1 - z - z^2} = \sum_{n=1}^{\infty} F_n z^n$$

Roots $\qquad \left(z + \frac{1+\sqrt{5}}{2}\right)\left(z - \frac{-1+\sqrt{5}}{2}\right)$



PARTIAL FRACTION DECOMPOSITION

$$\frac{z}{1-z-z^2} = \frac{1}{\sqrt{5}}\left\{\frac{1}{1-\frac{1+\sqrt{5}}{2}z} - \frac{1}{1-\frac{1-\sqrt{5}}{2}z}\right\}$$

GEOMETRIC SERIES

$$= \frac{1}{\sqrt{5}}\sum_{n=1}^{\infty}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\} z^n$$

RADIUS OF CONVERGENCE

$$= \sum_{n=1}^{\infty} F_n z^n \qquad |z| < \frac{2}{1+\sqrt{5}} = \frac{\sqrt{5}-1}{2} = R$$

CONCLUSION:

Small as n is large

$$\boxed{F_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}}$$

$(1{,}618)^n \qquad (-0{,}618)^n$

A. de Moivre 1718

= the nearest integer to

$$\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$$

$n = 1, 2, 3, 4, \ldots$

This is the n^{th} Fibonacci number!