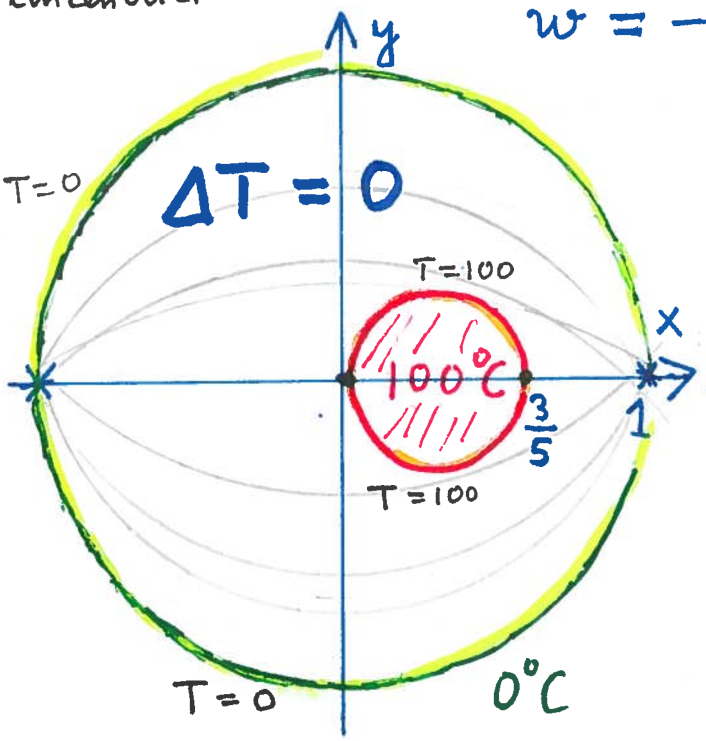
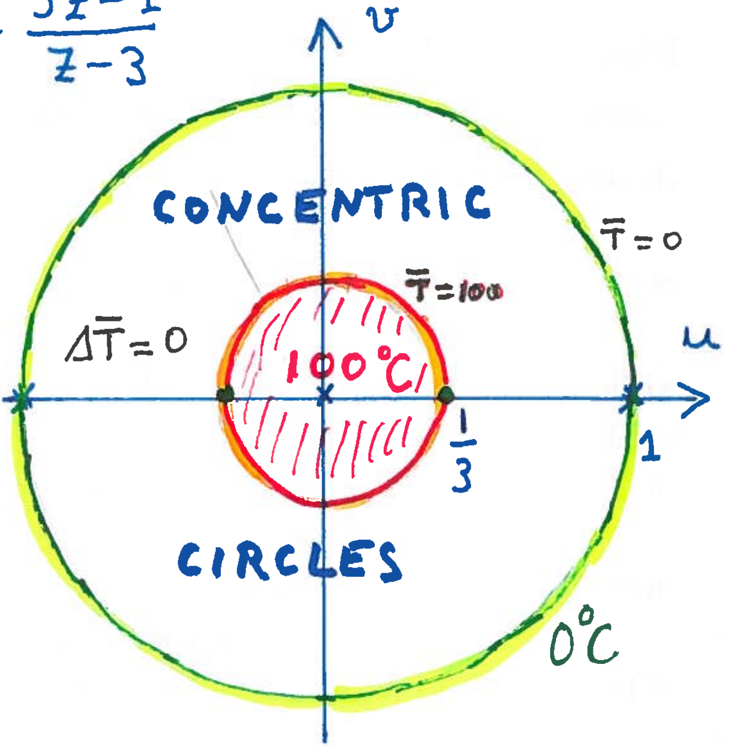
# STATIONARY TEMPERATURE

$z = x + iy$ $\qquad\qquad$ $w = u + iv$

Two infinite pipes, not concentric.

$$w = -\frac{3z-1}{z-3}$$




$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \overset{*)}{\Longleftrightarrow} \frac{\partial^2 \bar{T}}{\partial u^2} + \frac{\partial^2 \bar{T}}{\partial v^2} = 0$$

Change of coordinates:

$$T(x,y) = \bar{T}(u,v)$$

Radial solution

distance to the origin.

$$\bar{T}(u,v) = A \ln\left(\sqrt{u^2+v^2}\right) + B$$

$$= \underbrace{-\frac{100^\circ}{\ln(9)} \ln\left(u^2+v^2\right)}_{\text{Solves the concentric problem.}}$$

The conformal $^{*)}$ mapping

$$\boxed{w = -\frac{3z-1}{z-3}}$$

(MÖBIUS TR.)

Verify!: $|z| = 1 \Longleftrightarrow |w| = 1$ $\qquad 0^\circ$

Verify! $|z - 0{,}3| = 0{,}3 \Longleftrightarrow |w| = \frac{1}{3}$ $\qquad 100^\circ$

The solutions of Laplace's Equation are preserved under conformal mappings*). Now

$$w = -\frac{3z-1}{z-3} \qquad w = u + iv$$

$$u^2 + v^2 = \frac{(3x-1)^2 + 9y^2}{(x-3)^2 + y^2}$$

Temp. between the pipes.

ANSWER:

$$\boxed{T(x,y) = -\frac{100^\circ}{\ln(9)} \ln\left(\frac{(3x-1)^2 + 9y^2}{(x-3)^2 + y^2}\right)}$$

<u>Remark</u>: A Möbius transformation

$$w = \frac{az+b}{cz+d} \qquad (ad \neq bc)$$

maps circles to circles (or lines).

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*) The proof of the conformal invariance uses the Cauchy-Riemann equations $u_x = v_y$, $u_y = -v_x$ and the chain rule. Skipped here.

†) Require that

$$z = \left\{\begin{array}{l} 1 \rightarrow 1 \\ -1 \rightarrow -1 \\ 0 \rightarrow -\mathcal{H} \\ 3/5 \rightarrow +\mathcal{H} \end{array}\right\} = w$$

in $w = \dfrac{az+b}{cz+d}$. Then determine $\mathcal{H}$.