PROBLEM SET 6.3

1. **Report on Shifting Theorems.** Explain and compare the different roles of the two shifting theorems, using your own formulations and simple examples. Give no proofs.

2–11 SECOND SHIFTING THEOREM, UNIT STEP FUNCTION

Sketch or graph the given function, which is assumed to be zero outside the given interval. Represent it, using unit step functions. Find its transform. Show the details of your work.

2. $t \quad (0 < t < 2)$
3. $t - 3 \quad (t > 3)$
4. $\cos 2t \quad (0 < t < \pi)$
5. $e^{-t} \quad (0 < t < \pi)$
6. $\sin \pi t \quad (2 < t < 4)$
7. $e^{-\pi/2t} \quad (1 < t < 3)$
8. $t^2 \quad (1 < t < 2)$
9. $2t^2 \quad (t > \frac{5}{2})$
10. $\sinh t \quad (0 < t < 2)$
11. $\sin t \quad (\pi/2 < t < \pi)$

12–17 INVERSE TRANSFORMS BY THE 2ND SHIFTING THEOREM

Find and sketch or graph $f(t)$ if $\mathcal{L}(f)$ equals

12. $e^{-2s}/(s-1)^3$
13. $4(1 - e^{-\pi s})/(s^2 + 4)$
14. $4(e^{-2s} - 2e^{-5s})/s$
15. e^{-2s}/s^6
16. $2(e^{-s} - e^{-3s})/(s^2 - 4)$
17. $(1 + e^{-2\pi(s+1)})(s+1)/((s+1)^2 + 1)$

18–27 IVPs, SOME WITH DISCONTINUOUS INPUT

Using the Laplace transform and showing the details, solve

18. $4y'' - 12y' + 9y = 0, \quad y(0) = 2/3, y'(0) = 1$
19. $y'' - 6y' + 8y = e^{-t} - e^{-4t}, \quad y(0) = 1, y'(0) = 4$
20. $y'' + 10y' + 24y = 144t^2, \quad y(0) = 19/12, y'(0) = -5$
21. $y'' + 4y = 4 \cos t$, if $0 < t < \pi$, and 0 if $t > \pi$
22. $y'' + 3y' + 2y = 4t$ if $0 < t < 1$ and 8 if $t > 1$; $y(0) = 0, y'(0) = 0$
23. $y'' + y' - 2y = 3 \sin t - \cos t, (0 < t < 2\pi)$, and $3 \sin 2t - \cos 2t, (t > 2\pi); \quad y(0) = 0, y'(0) = -1$
24. $y'' + 3y' + 2y = 1$ if $0 < t < 1$ and 0 if $t > 1$; $y(0) = 0, y'(0) = 0$
25. $y'' + y = 2t$ if $0 < t < 1$ and 2 if $t > 1$
26. **Shifted data.** $y'' + 2y' + 5y = 10 \sin t$ if $0 < t < 2\pi$ and 0 if $t > 2\pi; \quad y(\pi) = 1, y'(\pi) = 2e^{-\pi} - 2$
27. **Shifted data.** $y'' + 4y = 8t^2$ if $0 < t < 5$ and 0 if $t > 5; \quad y(1) = 1 + \cos 2, y'(1) = 4 - 2 \sin 2$

28–40 MODELS OF ELECTRIC CIRCUITS

28–30 *RL*-CIRCUIT

Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. 126, assuming $i(0) = 0$ and:

28. $R = 1$ kΩ $(= 1000\ \Omega)$, $L = 1$ H, $v = 0$ if $0 < t < \pi$, and 40 sin t V if $t > \pi$
29. $R = 25\ \Omega$, $L = 0.1$ H, $v = 490\, e^{-5t}$ V if $0 < t < 1$ and 0 if $t > 1$
30. $R = 10\ \Omega$, $L = 0.5$ H, $v = 200t$ V if $0 < t < 2$ and 0 if $t > 2$

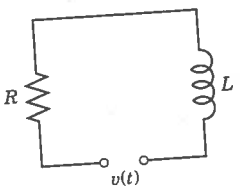

Fig. 126. Problems 28–30

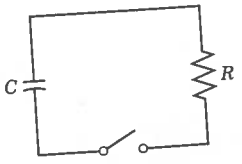

Fig. 127. Problem 31

31. **Discharge in *RC*-circuit.** Using the Laplace transform, find the charge $q(t)$ on the capacitor of capacitance C in Fig. 127 if the capacitor is charged so that its potential is V_0 and the switch is closed at $t = 0$.

32–34 *RC*-CIRCUIT

Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. 128 with $R = 10\ \Omega$ and $C = 10^{-2}$ F, where the current at $t = 0$ is assumed to be zero, and:

32. $v = 0$ if $t < 4$ and $14 \cdot 10^6 e^{-3t}$ V if $t > 4$
33. $v = 0$ if $t < 2$ and $100(t - 2)$ V if $t > 2$
34. $v(t) = 100$ V if $0.5 < t < 0.6$ and 0 otherwise. Why does $i(t)$ have jumps?

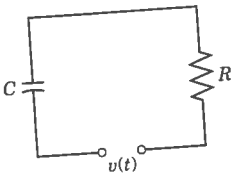

Fig. 128. Problems 32–34

35–37 *LC*-CIRCUIT

Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. 129, assuming zero initial current and charge on the capacitor and:

35. $L = 1$ H, $C = 10^{-2}$ F, $v = -9900 \cos t$ V if $\pi < t < 3\pi$ and 0 otherwise
36. $L = 1$ H, $C = 0.25$ F, $v = 200\,(t - \frac{1}{3}t^3)$ V if $0 < t < 1$ and 0 if $t > 1$
37. $L = 0.5$ H, $C = 0.05$ F, $v = 78 \sin t$ V if $0 < t < \pi$ and 0 if $t > \pi$

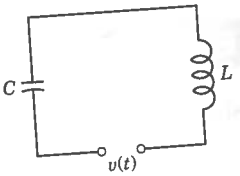

Fig. 129. Problems 35–37

38–40 *RLC*-CIRCUIT

Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. 130, assuming zero initial current and charge and:

38. $R = 4\ \Omega$, $L = 1$ H, $C = 0.05$ F, $v = 34e^{-t}$ V if $0 < t < 4$ and 0 if $t > 4$