PROBLEM SET 6.2

1–11 INITIAL VALUE PROBLEMS (IVPS)

Solve the IVPs by the Laplace transform. If necessary, use partial fraction expansion as in Example 4 of the text. Show all details.

1. $y' + \frac{2}{3}y = -4\cos 2t, \quad y(0) = 0$
2. $y' + 2y = 0, \quad y(0) = 1.5$
3. $y'' + y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = 1$
4. $y'' + 9y = 10e^{-t}, \quad y(0) = 0, \quad y'(0) = 0$
5. $y'' - \frac{1}{4}y = 0, \quad y(0) = 12, \quad y'(0) = 0$
6. $y'' - 6y' + 5y = 29\cos 2t, \quad y(0) = 3.2,$ $y'(0) = 6.2$
7. $y'' + 7y' + 12y = 21e^{3t}, \quad y(0) = 3.5,$ $y'(0) = -10$
8. $y'' - 4y' + 4y = 0, \quad y(0) = 8.1, \quad y'(0) = 3.9$
9. $y'' - 3y' + 2y = 4t - 8, \quad y(0) = 2, \quad y'(0) = 7$
10. $y'' + 0.04y = 0.02t^2, \quad y(0) = -25, \quad y'(0) = 0$
11. $y'' + 3y' + 2.25y = 9t^3 + 64, \quad y(0) = 1,$ $y'(0) = 31.5$

12–15 SHIFTED DATA PROBLEMS

Solve the shifted data IVPs by the Laplace transform. Show the details.

12. $y'' + 2y' - 3y = 0, \quad y(2) = -3,$ $y'(2) = -5$
13. $y' - 6y = 0, \quad y(-1) = 4$
14. $y'' + 2y' + 5y = 50t - 100, \quad y(2) = -4,$ $y'(2) = 14$
15. $y'' + 3y' - 4y = 6e^{2t-3}, \quad y(1.5) = 4,$ $y'(1.5) = 5$

16–21 OBTAINING TRANSFORMS BY DIFFERENTIATION

Using (1) or (2), find $\mathcal{L}(f)$ if $f(t)$ equals:

16. $t\cos 4t$
17. te^{at}
18. $\cos^2 2t$
19. $\cos^2 \omega t$
20. $\sin^4 t$. Use Prob. 19.
21. $\sinh^2 t$

22. **PROJECT. Further Results by Differentiation.** Proceeding as in Example 1, obtain

$$\text{(a)}\quad \mathcal{L}(t\cos\omega t) = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

and from this and Example 1: **(b)** formula 21, **(c)** 22, **(d)** 23 in Sec. 6.9,

$$\text{(e)}\quad \mathcal{L}(t\cosh at) = \frac{s^2+a^2}{(s^2-a^2)^2},$$

$$\text{(f)}\quad \mathcal{L}(t\sinh at) = \frac{2as}{(s^2-a^2)^2}.$$

23–29 INVERSE TRANSFORMS BY INTEGRATION

Using Theorem 3, find $f(t)$ if $\mathcal{L}(F)$ equals:

23. $\dfrac{2}{s^2 + s/3}$
24. $\dfrac{20}{s^3 - 2\pi s^2}$
25. $\dfrac{1}{s(s^2 + \omega^2/4)}$
26. $\dfrac{1}{s^4 - s^2}$
27. $\dfrac{s+8}{s^4 + 4s^2}$
28. $\dfrac{3s+4}{s^4 + k^2s^2}$
29. $\dfrac{1}{s^3 + as^2}$

30. **PROJECT. Comments on Sec. 6.2. (a)** Give reasons why Theorems 1 and 2 are more important than Theorem 3.

(b) Extend Theorem 1 by showing that if $f(t)$ is continuous, except for an ordinary discontinuity (finite jump) at some $t = a$ (>0), the other conditions remaining as in Theorem 1, then (see Fig. 117)

$$\text{(1*)}\quad \mathcal{L}(f') = s\mathcal{L}(f) - f(0) - [f(a+0) - f(a-0)]e^{-as}.$$

(c) Verify (1*) for $f(t) = e^{-t}$ if $0 < t < 1$ and 0 if $t > 1$.

(d) Compare the Laplace transform of solving ODEs with the method in Chap. 2. Give examples of your own to illustrate the advantages of the present method (to the extent we have seen them so far).

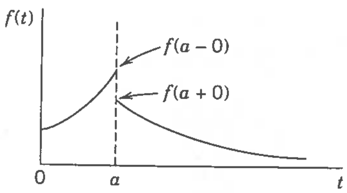

Fig. 117. Formula (1*)