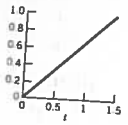
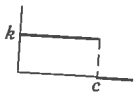
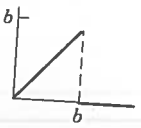
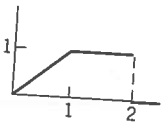
## 1–16 LAPLACE TRANSFORMS

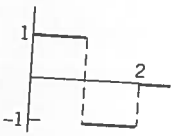
Find the transform. Show the details of your work. Assume that $a$, $b$, $\omega$, $\theta$ are constants.

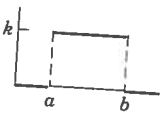
1. $2t + 8$
2. $(a - bt)^2$
3. $\cos 2\pi t$
4. $\cos^2 \omega t$
5. $e^{3t} \sinh t$
6. $e^{-t} \sinh 4t$
7. $\cos(\omega t + \theta)$
8. $1.5 \sin(3t - \pi/2)$
9. 

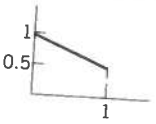
10. 

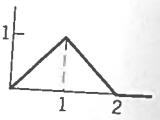
11. 

12. 

13. 

14. 

15. 

16. 


## 17–24 SOME THEORY

17. **Table 6.1.** Convert this table to a table for finding inverse transforms (with obvious changes, e.g., $\mathcal{L}^{-1}(1/s^n) = t^{n-1}/(n-1)$, etc).
18. Using $\mathcal{L}(f)$ in Prob. 10, find $\mathcal{L}(f_1)$, where $f_1(t) = 0$ if $t \leqq 2$ and $f_1(t) = 1$ if $t > 2$.
19. **Table 6.1.** Derive formula 6 from formulas 9 and 10.
20. **Nonexistence.** Show that $e^{t^2}$ does not satisfy a condition of the form (2).
21. **Nonexistence.** Give simple examples of functions (defined for all $t \geqq 0$) that have no Laplace transform.
22. **Existence.** Show that $\mathcal{L}(1/\sqrt{t}) = \sqrt{\pi}/s$. [Use (30) $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ in App. 3.1.] Conclude from this that the conditions in Theorem 3 are sufficient but not necessary for the existence of a Laplace transform.



23. **Change of scale.** If $\mathcal{L}(f(t)) = F(s)$ and $c$ is any positive constant, show that $\mathcal{L}(f(ct)) = F(s/c)/c$ (*Hint:* Use (1).) Use this to obtain $\mathcal{L}(\cos \omega t)$ from $\mathcal{L}(\cos t)$.
24. **Inverse transform.** Prove that $\mathcal{L}^{-1}$ is linear. *Hint:* Use the fact that $\mathcal{L}$ is linear.

## 25–32 INVERSE LAPLACE TRANSFORMS

Given $F(s) = \mathcal{L}(f)$, find $f(t)$. $a$, $b$, $L$, $n$ are constants. Show the details of your work.

25. $\dfrac{0.2s + 1.4}{s^2 + 1.96}$
26. $\dfrac{5s + 1}{s^2 - 25}$
27. $\dfrac{s}{L^2 s^2 + 1/4\,\pi^2}$
28. $\dfrac{1}{(s + \sqrt{2})(s - \sqrt{3})}$
29. $\dfrac{2}{s^4} - \dfrac{48}{s^6}$
30. $\dfrac{4s + 32}{s^2 - 16}$
31. $\dfrac{-s + 11}{s^2 - 2s - 3}$
32. $\dfrac{1}{(s + a)(s + b)}$

## 33–45 APPLICATION OF $s$-SHIFTING

In Probs. 33–36 find the transform. In Probs. 37–45 find the inverse transform. Show the details of your work.

33. $t^3 e^{-2t}$
34. $ke^{-at} \cos \omega t$
35. $2e^{-1/2t} \sin 4\pi t$
36. $\sinh t \cos t$
37. $\dfrac{2\pi}{(s + \pi)^3}$
38. $\dfrac{6}{(s + 1)^3}$
39. $\dfrac{90}{(s + \sqrt{3})^6}$
40. $\dfrac{4}{s^2 - 2s - 3}$
41. $\dfrac{\pi}{s^2 + 4s\pi + 3\pi^2}$
42. $\dfrac{a_0}{s + 1} + \dfrac{a_1}{(s + 1)^2} + \dfrac{a_2}{(s + 1)^3}$
43. $\dfrac{6s + 7}{2s^2 + 4s + 10}$
44. $\dfrac{a(s + k) + b\pi}{(s + k)^2 + \pi^2}$
45. $\dfrac{k_0}{s} + \dfrac{k_1}{(s - a)^2}$