Problem 1

$$\frac{e^{-2s}}{s^2} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s^2} - \frac{e^{-3s}}{s}$$

Problem 2

$$\frac{1}{5}e^{-t} + \frac{4}{5}e^{t}\cos t - \frac{3}{5}e^{t}\sin t + u(t-1)e^{t-1}\sin(t-1).$$

Problem 3

a)

$$X(s) = \frac{2}{(s-1)^2-4},\ Y(s) = \frac{2(s-1)}{(s-1)^2-4}$$

b)

$$x(t) = \frac{1}{2}e^{3t} - \frac{1}{2}e^{-t},\ y(t) = e^{3t} + e^{-t}.$$

Problem 4

$$y(t) = -\frac{1}{2}t + e^{t}\cos t + \frac{1}{2}e^{t}\sin t.$$

Problem 5

a)

$$b_n = \frac{2}{n\pi}(-1)^{n+1} - \frac{4(1-(-1)^n)}{n^3\pi^3}, \quad S_f(0) = 0;\ S_f(\frac{1}{2}) = \frac{1}{4},\ S_f(1) = 0.$$

b)

$$a_0 = \frac{1}{3},\ a_n = \frac{4}{(n\pi)^2}(-1)^n,$$

c) I would use the cos series. Its coefficients decay faster because $C_f(x)$ is a continuous function. In order to have error about 10^{-5} I would use N about 15. $E_N^2 \sim \frac{1}{2}\sum_{n>N}\frac{16}{n^4\pi^4} \sim \frac{8}{3\pi^4N^3}$.

Problem 6

a)

$$\sum_{n=-\infty}^{\infty} c_n e^{2inx};\ c_0 = \frac{\pi^3}{12};\ c_n = -\frac{1}{4}\frac{\pi}{n^2} + i\frac{3}{4n^3}.$$

$$\frac{\pi^3}{12} - \frac{1}{2}\sum_{1}^{\infty}\frac{\pi}{n^2}\cos 2nx; \quad -\sum_{1}^{\infty}\frac{3}{2n^3}\sin 2nx;$$

b)

$$y(x) = \sum_{-\infty} \infty \frac{c_n}{-4n^2 + 6in + 2} e^{2inx},$$

where c_n are defined in **a)**.

Problem 7

a) $(f(x-a))\hat{}(w) = e^{-iwa}\hat{f}(w)$.

b)

$$\frac{1}{\sqrt{2}} e^{p^2} e^{-ipw} e^{-w^2/4}.$$

Problem 8

a)

$$\hat{f}(w) = \sqrt{\frac{2}{\pi}} \frac{\sin w}{w}.$$

b)

$$h(x) = \begin{cases} x+2, & -2 < x < 0 \\ 2-x, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

c)

$$\hat{h}(w) = \frac{2\sqrt{2}}{\sqrt{\pi}} \left(\frac{\sin w}{w} \right)^2$$

Problem 9

a) $u(t,x) = \sin nx (A_n \cos cnt + B_n \sin cnt)$

b) $u(x,t) = c^{-1} \sin x \sin ct + (4c)^{-1} \sin 2x \sin 2ct$

Problem 10

$\hat{u}(w,t) = e^{-c^2 w^2 t} \hat{u}(w,0)$ and $\hat{u}(w,0) = -iw/(2\sqrt{2}) e^{-w^2/4}$,

$$u(x,t) = \mathcal{F}^{-1}(-\frac{i}{2\sqrt{2}} w e^{-(1/4+c^2 t)w^2}$$

We use the formula from the hint to compute the inverse Fourier transform, take a such that $1/4 + c^2 t = 1/(4a)$ and get

$$u(x,t) = (4c^2 t + 1)^{-3/2} \exp(-x^2/(1+4c^2 t))$$