REKKER: $\sum_{n=0}^{\infty} b_n \quad (b_n \in \mathbb{C} \ \forall n \in \mathbb{N})$

•) $\sum_{n=0}^{\infty} b_n$ KONV $\Rightarrow \lim_{n\to\infty} b_n = 0$

$\lim b_n \neq 0$ ELLER EKSISTERER IKKE $\Longrightarrow \sum_{n=0}^{\infty} b_n$ DIV.

•) $\sum_{n=0}^{\infty} b_n$ KONV OG $\begin{cases} \sum_{n=0}^{\infty} |b_n| \text{ DIV} \Rightarrow \sum_{n=0}^{\infty} b_n \text{ BETINGET KONV} \\ \sum_{n=0}^{\infty} |b_n| \text{ KONV} \Rightarrow \sum_{n=0}^{\infty} |b_n| \text{ ABSOLUTT KONV.} \end{cases}$

•) $\sum_{n\in\mathbb{N}} b_n$, $b_n \neq 0 \ \forall n \in \mathbb{N}$:

•) $\lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| = q < 1 \Rightarrow \sum_{n=0}^{\infty} b_n$ KONV. (ABS)

•) $\lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| = 1 \Rightarrow$ VET IKKE.

•) $\exists N \in \mathbb{N}$ SLIK AT $\left|\frac{b_{n+1}}{b_n}\right| = 1 \ \forall n \in \mathbb{N}$ MED $n \geq N$
$\Rightarrow \sum_{n=0}^{\infty} b_n$ DIV.

•) $\lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| = q > 1 \Rightarrow \sum_{n=0}^{\infty} b_n$ DIV.

•) $\sum_{n=0}^{\infty} b_n$

•) $\lim_{n\to\infty} \sqrt[n]{|b_n|} = q < 1 \Rightarrow \sum_{n=0}^{\infty} b_n$ KONV (ABS)

•) $\lim_{n\to\infty} \sqrt[n]{|b_n|} = 1 \Rightarrow$ VET IKKE.

•) $\exists N \in \mathbb{N}$ SLIK AT $\sqrt[n]{|b_n|} = 1 \ \forall n \geq N \Rightarrow \sum b_n$ DIV

•) $\lim_{n\to\infty} \sqrt[n]{|b_n|} = q > 1 \Rightarrow \sum_{n=0}^{\infty} b_n$ DIV.

POTENSREKKER: $\sum_{n=0}^{\infty} a_n(z-z_0)^n$

KONV FOR $z_1 \neq z_0$

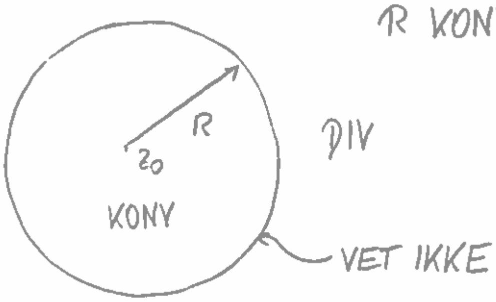
DIV FOR $z_2 \neq z_0$.

KONV FOR $z=z_0$

ABS KONV FOR $\forall z \in \mathbb{C}$ MED $|z-z_0| < |z_0-z_1|$

DIV FOR $\forall z \in \mathbb{C}$ MED $|z-z_0| > |z_2-z_0|$

R KONVERGENSRADIUS



$$R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$$

$$R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{|a_n|}}$$

$$\sum_{n=0}^{\infty} a_n (z-z_0)^n \quad \forall |z-z_0|<R$$

$$f: \{z\in\mathbb{C} \mid |z-z_0|<R\} \to \mathbb{C}$$
$$z \mapsto f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n$$

$$\sum_{n=0}^{\infty} a_n (z-z_0)^n = \sum_{n=0}^{\infty} b_n (z-z_0)^n \quad \forall |z-z_0|<R$$
$$\Downarrow$$
$a_n = b_n$ FOR $n = 0, 1, 2, \ldots$

KOEFF. a_n ENTYDIG BESTEMT

f KONT I z_0

$$\sum_{n=1}^{\infty} n a_n (z-z_0)^{n-1} \quad \forall |z-z_0|<R$$
$$\sum_{n=0}^{\infty} \frac{a_n}{n+1} (z-z_0)^{n+1} \quad \forall |z-z_0|<R$$

f ANALYTISK

$$f': \{z\in\mathbb{C} \mid |z-z_0|<R\} \to \mathbb{C}$$
$$z \mapsto f'(z) = \sum_{n=1}^{\infty} n a_n (z-z_0)^{n-1}$$

$$f^{(n)}: \{z\in\mathbb{C} \mid |z-z_0|<R\} \to \mathbb{C}$$
$$z \mapsto f^{(n)}(z) = \ldots\ldots$$

n GANGER TERMVIS DERIVASJON

OBS: $\sum_{n=0}^{\infty} a_n (z-z_0)^n \ \forall |z-z_0|<R_1, \ \sum_{n=0}^{\infty} b_n (z-z_0)^n \ \forall |z-z_0|<R_2$

$$\Rightarrow \sum_{n=0}^{\infty} (a_n+b_n)(z-z_0)^n = \sum_{n=0}^{\infty} a_n (z-z_0)^n + \sum_{n=0}^{\infty} b_n (z-z_0)^n \quad \forall |z-z_0| < \min(R_1,R_2)$$

$$\sum_{n=0}^{\infty} \left(\sum_{i=0}^{n} a_i b_{n-i}\right)(z-z_0)^n = \left(\sum_{n=0}^{\infty} a_n (z-z_0)^n\right)\left(\sum_{n=0}^{\infty} b_n (z-z_0)^n\right) \quad \forall |z-z_0| < \min(R_1,R_2).$$

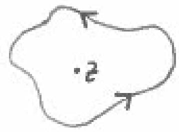
$f: D \to \mathbb{C}$ ANALYTISK

D... ENKELTSAMMENHENGENDE DOMENE

$$f(z) = \frac{1}{2\pi i} \oint_C \frac{f(z^*)}{z^* - z}\, dz^*$$

$\forall$ SIMPEL LUKKETE KURVER C I D MED

$z_0 \in D$



$$f(z) = \sum_{k=0}^{n} a_k (z-z_0)^k + R_n(z)$$

$$a_n = \frac{1}{n!} f^{(n)}(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z^*)}{(z^*-z_0)^{n+1}}\, dz^*$$

$$R_n(z) = \frac{(z-z_0)^{n+1}}{2\pi i} \oint_C \frac{f(z^*)}{(z^*-z_0)^{n+1}(z^*-z)}\, dz^*$$

$n \to \infty$

TAYLORREKKE:

$$f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n \quad \forall |z-z_0| < R.$$

R... RADIUS TIL DEN STØRSTE SIRKELSKIVEN I D MED SENTRUM I z_0

EN POTENSREKKE $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ MED KONV.RADIUS $R>0$ ER LIK TAYLORREKKEN TIL DENS SUMME MED SENTRUM z_0.